

B.01 Controllability, observability, and stabilizability

The three topics controllability, observability, and stabilizability are three topics of central concern to linear systems theory.

Controllability

Controllability is defined as follows.

Controllability

Definition B.1: controllable and uncontrollable

If there exists some input to a linear system such that any initial state in its state space can be evolved in finite time to any final state in its state space, the system is *controllable*. Otherwise, the system is *uncontrollable*.

A given system's controllability can be determined from the following.

Definition B.2: controllability matrix

Let a linear system of order n and number of inputs r have state space $\{A, B, C, D\}$. We define the $n \times nr$ *controllability matrix* to be

$$U = [B \mid AB \mid A^2B \mid \dots \mid A^{n-1}B].$$

The following well-known theorem, left unproven here, allows us to easily determine the controllability of a given system.

Theorem B.3: controllability

A linear system is controllable if its controllability matrix has full rank. If it is less than full rank, the linear system is uncontrollable.