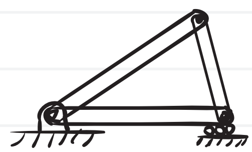
Introduction to 2D trusses

A structure is a set of connected rigid bodies.

A truss is a structure composed of lightweight rigid bodies connected only at their endpoints. Our study of structures begins with them.



Trusses are used to make many structures, including bridges, buildings, vehicles, and aircraft. In the cases of bridges and buildings, the members are typically made of steel. In the case of vehicles and aircraft, the members are often made of aluminum or carbon fiber for their lightness, but steel is also used when strength is needed or fatigue failure is problematic.

Members are joined in many ways. For bridges, a gusset plate is typically used. For many trusses, welding, bolts, or pins join the members. Most of the joining methods are rigid, leaving zero degrees of freedom.

Our modeling of trusses for analysis will include some important assumptions that are not strictly true in real trusses, but are approximately valid. They are, as follows.

1. External forces are applied only at joints.
2. If the applied loads are much larger than the weights of the members, the weight of each member is ignored. Otherwise, half of the weight of each member is applied at each of its joints.
3. The joints are all smooth pins.

These assumptions often yield accurate results, but must always be reconsidered for each truss.

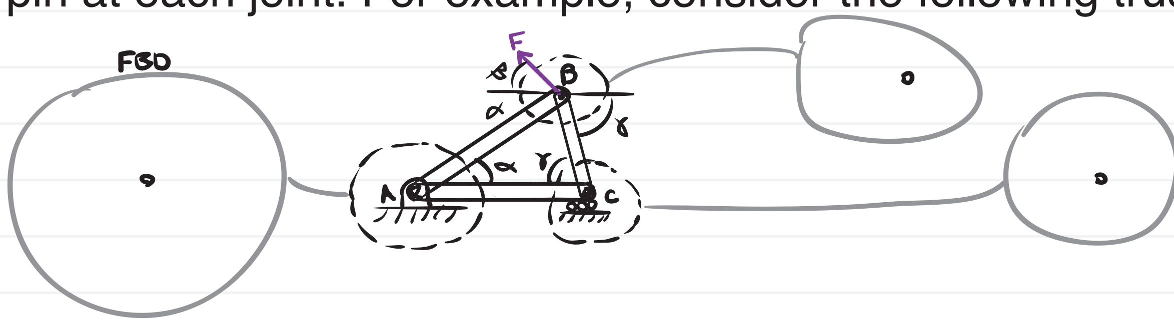
Under our assumptions, a very convenient simplification can be made in our analysis: since all members are two-force members, their forces must be colinear (along the same line). This leaves only two possibilities for the loading of each member: tension or compression.

The sign convention we will adopt is that we will assign tension for each member to be the positive direction of an unknown member force (so we draw the arrow in a free body diagram thusly).

There are two primary methods used for analyzing trusses. The first, the method of joints, will be explored here. Later, we will consider the method of sections.

The method of joints

If the structure is in equilibrium, we know that every joint in the structure is also at equilibrium. So we can apply our equilibrium equations to the pin at each joint. For example, consider the following truss.



As we have shown, we draw free body diagrams (FBDs) of each pin. We can then apply the equilibrium equations to each pin.

$$\xrightarrow{x} \Sigma F = 0, \quad y\uparrow \Sigma F = 0, \quad \text{and} \quad \Sigma M_{pin} = 0.$$

However, the moment equation is trivial: all forces in each FBD pass through the pin! So we really have only the two force equations.

So, in our example, for each pin, we can write the equilibrium equations as follows.

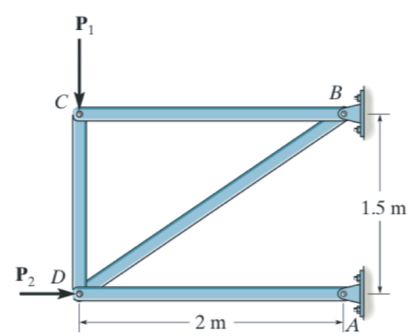
	$\xrightarrow{x} \Sigma F = 0$	$y\uparrow \Sigma F = 0$
A	$F_{Ax} + F_{AC} + F_{AB}\cos\alpha = 0$	$F_{Ay} + F_{AB}\sin\alpha = 0$
B	$F_{BC}\cos\gamma - F\cos\beta - F_{AB}\cos\alpha = 0$	$F\sin\beta - F_{AB}\sin\alpha - F_{BC}\sin\gamma = 0$
C	$-F_{AC} - F_{BC}\cos\gamma = 0$	$F_C + F_{BC}\sin\gamma = 0$

Let's assume that the applied force **F** and all angles are given. Three reaction forces and three member forces are unknown, six in total. There are also six independent equations, so we can solve for all unknown forces.

A common way to proceed is to find joints with only two unknown forces and solve their equilibrium equations first, before moving on. This is tantamount to solving subsets of the system of equations, first.

Example: Hibbeler (2015) Problem 6-2

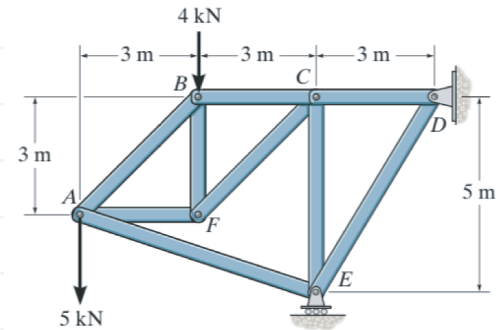
6–2. Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 45$ kN, $P_2 = 30$ kN.



Solution

Example: Hibbeler (2015) Problem 6-7

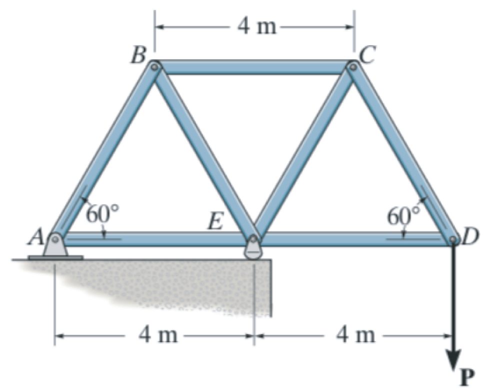
6–7. Determine the force in each member of the truss and state if the members are in tension or compression.



Solution

Example: Hibbeler (2015) Problem 6-17

6–17. If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force *P* that can be supported at joint *D*.



Solution