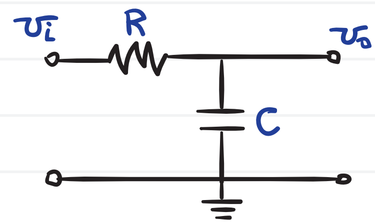
First-order filter response

A passive first-order filter is an example of a measurement system (a signal conditioner) that is first-order with a (typically) sinusoidal forcing function.

Low-pass filter response



The RC-circuit in the figure is often used as a low-pass filter.

Circuit analysis

Response analysis

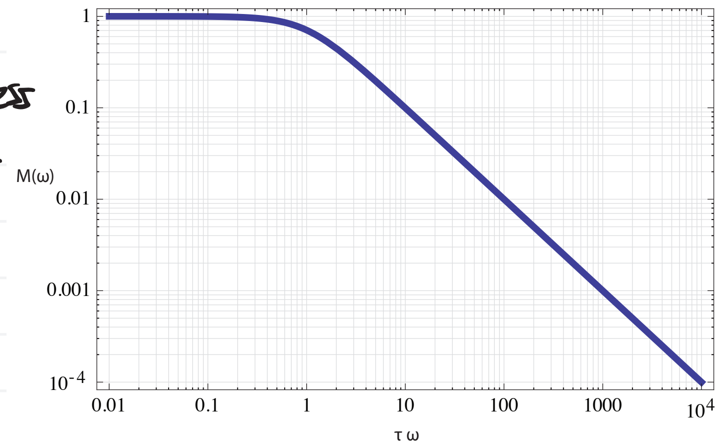
This is a first-order ODE. If the circuit is being used as a filter, we usually characterize it in terms of its magnitude ratio $M(\omega)$ and phase ϕ, which relate the magnitude and phase of an input sinusoid to the corresponding magnitude and phase of the output sinusoid.

We already did this analysis, which gave that

$$M(\omega) \equiv \frac{\text{amplitude of s.s. output}}{\text{amplitude of forcing func.}} = \frac{1}{\sqrt{1+\tau^2\omega^2}} \quad \text{and} \quad \phi \equiv -\operatorname{atan}(\tau\omega),$$

where we identify $\tau = RC$ as the time constant.

The plots show $M(\omega)$ and $\phi(\omega)$. $\tau\omega$ is the dimensionless variable often used in these plots



$\omega_c \equiv \frac{1}{\tau}$ is called the cutoff frequency. At this frequency, the magnitude ratio is -3 dB. dB (decibels) are related to amplitudes through the relation

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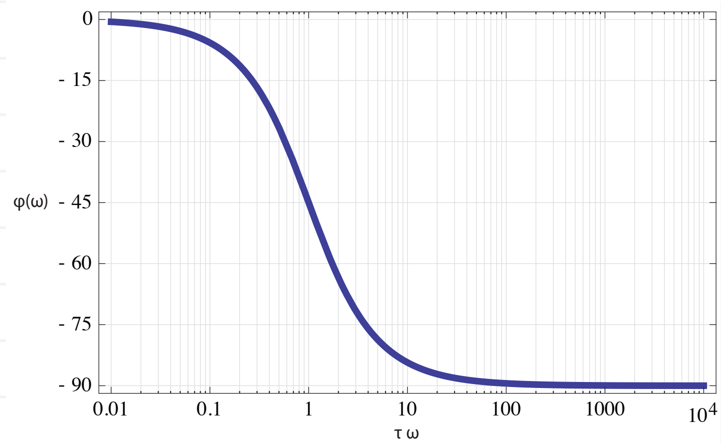
The -3dB point relates to an amplitude ratio, then, of



Notice that the magnitude ratio for $\omega < \omega_c$ is near 1, which means the output amplitude is nearly equal to the input amplitude. For $\omega > \omega_c$, the output amplitude is attenuated, and for $\omega >> \omega_c$, it is very small. For these reasons, this circuit is called a low-pass filter.

The graph for ϕ shows that the output lags the input by an amount between $0°$ and $90°$. A the cutoff frequency ω_c, $\phi(\omega_c) = -45°$.