Fourier series + orthogonal functions

where t_0 is some time, n is an integer, and T is the period of f).

The Fourier series allows us to write any periodic function as a sum of sinusoidal functions of various amplitudes. This series can be expressed as

, where

Fourier analysis is what we call finding A_0, A_n, + B_n for a given function $y(t)$.
Fourier synthesis is what we call finding $y(t)$ for a given set of amplitudes A_0, A_n, + B_n.

We will consider these in detail. First, it is helpful to consider the concept of orthogonal functions.

Orthogonal functions

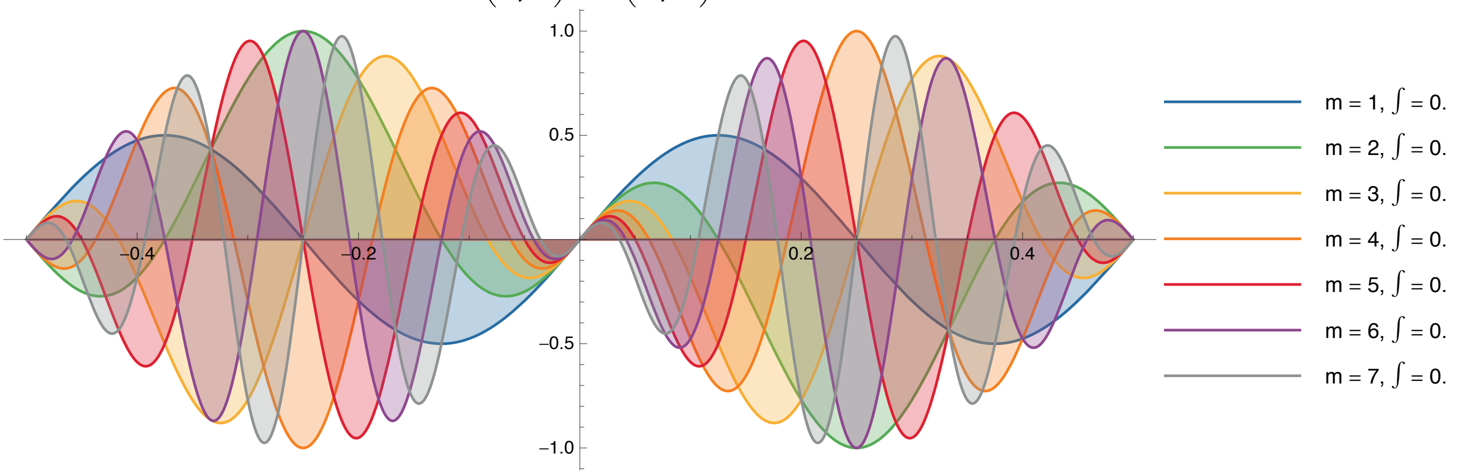
Two functions $x(t)$ and $y(t)$ are defined as orthogonal over the interval $a \le t \le b$ if the inner product over the interval

$$\int_{-T/2}^{T/2} \sin\left(\frac{2\pi n}{T}\tau\right)\ \cos\left(\frac{2\pi m}{T}\tau\right) d\tau = 0$$

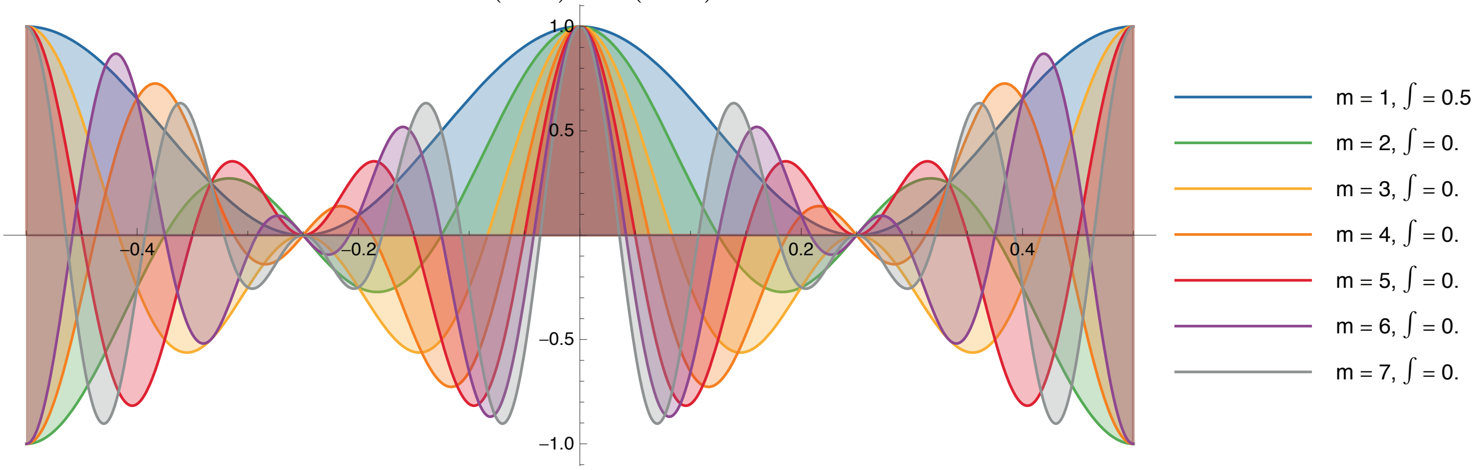
$$\int_{-T/2}^{T/2} \cos\left(\frac{2\pi n}{T}\tau\right)\ \cos\left(\frac{2\pi m}{T}\tau\right) d\tau = \begin{cases} 0 & m \neq n \\ \frac{T}{2} & m = n \neq 0 \\ T & m = n = 0 \end{cases}$$

$$\int_{-T/2}^{T/2} \sin\left(\frac{2\pi n}{T}\tau\right)\ \sin\left(\frac{2\pi m}{T}\tau\right) d\tau = \begin{cases} 0 & m \neq n \\ \frac{T}{2} & m = n \end{cases}$$

$\sin\left(\frac{2\pi n}{T}t\right)\ \cos\left(\frac{2\pi m}{T}t\right)$



$\cos\left(\frac{2\pi n}{T}t\right)\ \cos\left(\frac{2\pi m}{T}t\right)$



$\sin\left(\frac{2\pi n}{T}t\right)\ \sin\left(\frac{2\pi m}{T}t\right)$