Eigenvalues + eigenvectors

We will now recall the eigenvalue or eigenvector problem for an $n \times n$ matrix $A \in \mathbb{R}^n \times \mathbb{R}^n$, vector $\vec{m} \in \mathbb{R}^n$, and scalar $\lambda \in \mathbb{R}$:

(*)

Those values of $\lambda = \lambda_i$ and corresponding vectors $\vec{m} = \vec{m}_i$ for which (*) is satisfied are called eigenvalues (λ_i) and eigenvectors ($\vec{m}_i$).

Finding eigenvalues

Arranging (*),

(*')

For a nontrivial solution, this implies

$$\boxed{}\,, \qquad (**)$$

which is the characteristic equation for matrix A. If we expand the determinant, we get an n^{th}-order polynomial in λ:

$$\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0 = 0,$$

which can be solved for n roots. We will only consider the case of n distinct roots (no repetition!). These will be the eigenvalues.

Finding eigenvectors

Each eigenvalue has a corresponding eigenvector. We find the eigenvector $\vec{m}_i$ for an eigenvalue λ_i by substituting λ_i into (*) and solving for $\vec{m}$.

Example

Let $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A.

Eigenvalues — We use (**) :

Eigenvectors — Substitute each λ_i into (*′):