Transfer functions for SISO systems

Recall the SISO ODE for an n^{th} order system with input $u(t)$ and output $y(t)$,

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \dots + b_1 \frac{du}{dt} + b_0 u, \quad (*)$$

where $a_i \in \mathbb{R}$ and $b_i \in \mathbb{R}$ are constants.

Let the input be of the form (we will remove this restriction in subsequent chapters)

$$u(t) = U(s) e^{st},$$

where $U(s)$ is a complex-valued function and $s = \sigma + j\omega$ is a complex constant (with real part denoted σ and complex part denoted ω).

The method of undermined coefficients yields the assumed form of the particular solution to be

$$y_p(t) = Y(s) e^{st},$$

where $Y(s)$ is an unknown complex-valued function. Substituting $u(t)$ + $y(t)$ into $(*)$,

$$a_n \frac{d^n}{dt^n} Y(s) e^{st} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} Y(s) e^{st} + \dots + a_1 \frac{d}{dt} Y(s) e^{st} + a_0 Y(s) e^{st}$$

$$= b_m \frac{d^m}{dt^m} U(s) e^{st} + b_{m-1} \frac{d^{m-1}}{dt^{m-1}} U(s) e^{st} + \dots + b_1 \frac{d}{dt} U(s) e^{st} + b_0 U(s) e^{st} \Longrightarrow$$

$(**)$

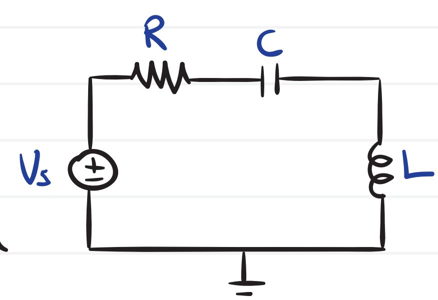
Definition: the system transfer function $H(s)$ is the output complex amplitude $Y(s)$ divided by the input complex amplitude $U(s)$. I.e.

Then, from (**),

We often represent this in a block diagram as follows.

Example The circuit shown has input-output ODE

$$L\frac{d^2 v_L}{dt^2} + R\frac{dv_L}{dt} + \frac{1}{C}v_L = L\frac{d^2 V_s}{dt^2}.$$



What is the transfer function between V_s and v_L?

Solution