System poles + zeros

Recall the SISO ODE for an n^{th} order system with input $u(t)$ and output $y(t)$,

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1}u}{dt^{m-1}} + \dots + b_1 \frac{du}{dt} + b_0 u,$$

where $a_i \in \mathbb{R}$ and $b_i \in \mathbb{R}$ are constants. Also recall that the transfer function for this system is

$$H(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}. \qquad (*)$$

The values of s for which $|H(s)| \to \infty$ are called the system poles.

The values of s for which $|H(s)| \to 0$ are called the system zeros.

We can re-write $(*)$, with $K \equiv \frac{b_m}{a_n}$, as

$$\boxed{},$$

where p_i are the poles and z_i are the zeros.

Since the poles + zeros define the transfer function (within a constant), they also define the SISO system.

It is now time to observe a crucial identity:

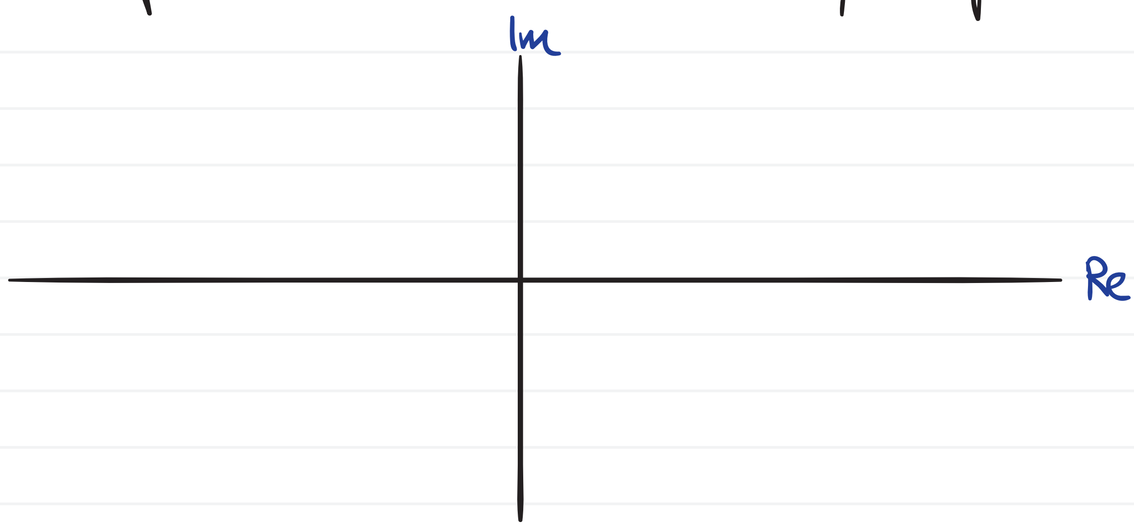
$$\boxed{}$$

Therefore, for systems with n distinct eigenvalues (poles), the homogeneous solution is

Pole-zero plots

Poles and zeros are either real (e.g. $p_i = \sigma_i$), imaginary pairs (e.g. $p_i = \pm j\omega_i$), or complex pairs (e.g. $p_i = \sigma_i \pm j\omega_i$).

We often plot these in the complex plane.



Stability

System poles describe system stability. Inspecting the homogeneous solution $y_h(t)$, we see that if the real part of all of a system's poles are negative, the response will decay to zero. The following situations are possible.