Transfer functions for MIMO + SISO systems — state space to transfer functions

In 005 notes, we established how to derive a transfer function from a SISO ODE. In this lecture, we will show how to derive a transfer function from a state space model, directly. The demonstration is valid for both SISO and MIMO systems.

In a manner analogous to that of the derivation of the system transfer operator $H\{\}$ from a state model, it can be shown that, for the state model

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} + D\vec{u},$$

the matrix of transfer functions is

[ ] .

$H(s)$ has the dimensions $m \times r$, where m is the number of outputs and r is the number of inputs. If $m=r=1$, we have a SISO system.

It turns out that each element of $H(s)$ has the same denominator $D(s) = \det(sI - A)$. I.e.:

$$H(s) = \begin{bmatrix} \frac{N_{11}(s)}{D(s)} & \cdots & \frac{N_{1r}(s)}{D(s)} \\ \vdots & \ddots & \\ \frac{N_{m1}(s)}{D(s)} & \cdots & \frac{N_{mr}(s)}{D(s)} \end{bmatrix}.$$

So all the transfer functions have the same poles, and differ only in zeros + scaling.

The term $H_{ij}(s)$ is the transfer function from input j to output i.