Lumped-parameter modeling of fluid systems

Power flow variables

The power flow $P(t)$ into a port of a fluid system is the product of the volumetric flow rate $Q(t)$ into the port and the pressure drop $P(t)$ across the port. I.e.

$$\qquad .$$

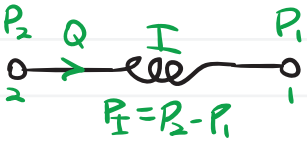
The total fluid volume is

$$V = \int_0^t Q(t)\,dt \quad .$$

The pressure momentum is

$$\Gamma = \int_0^t P(t)\,dt \quad .$$

Basic passive fluid elements

An ideal fluid inertance 

is an element for which

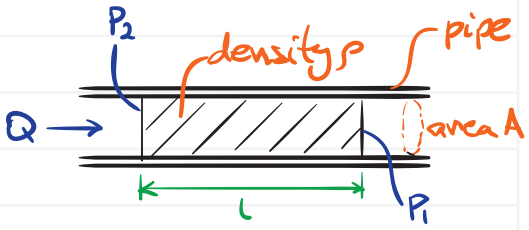
fluid inertance

$$\Gamma = I Q \quad .$$

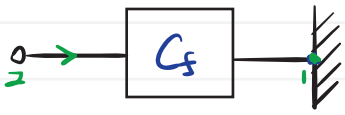
Differentiating both sides,

The fluid inertance (parameter) I depends on pipe geometry + fluid properties. For an incompressible fluid flowing in a pipe of uniform area A and length L,

with a uniform velocity profile, an element of fluid obey's Newton's law



An ideal fluid capacitance C_f defines the linear ideal fluid capacitor equation



$$V = C_f P \quad .$$

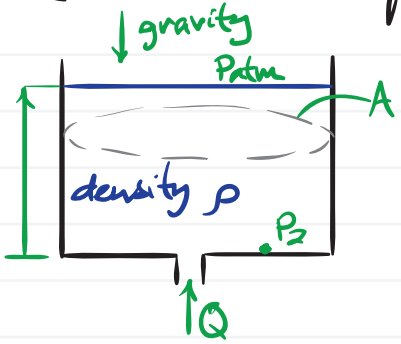
Differentiating w.r.t. time, as per usual:

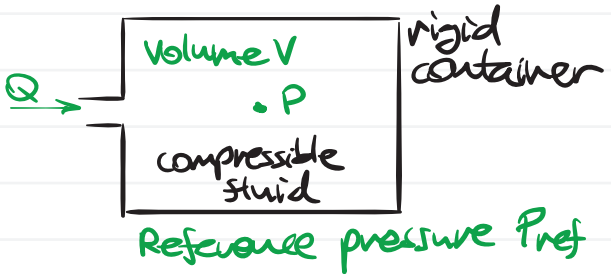
The fluid capacitance C_f can be derived for a tank of incompressible fluid of density ρ + cross-sectional area A to be

$$C_f = \frac{A}{\rho g} \quad .$$

Below are two examples of fluid capacitors.

tank in gravity field





An ideal fluid resistor 
has resistance R_f
and constitutive and elemental equation

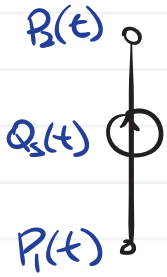
Elemental eq.

Pipes, orifices, and valves have fluid resistance.

Active fluid elements

There are pressure + flow sources, as shown below.

Flow source



Pressure source

$P_s = P_2 - P_1$

These are ideal sources that are assumed to always provide the prescribed flow or pressure, regardless of power demands.