Lumped-parameter modeling of thermal systems

The two power flow variables for thermal systems are heat flow rate q and temperature T. But, unlike in the other energy domains,

$$\mathcal{P}(t) \neq q\,T \,.$$ Difference 1

This is one of three ways thermal systems differ from the others we've considered. The other two are:
there is no T-type energy-storage element — Difference 2
& the D-type element does not dissipate energy. — Difference 3

The two types of passive thermal elements are thermal capacitors (A-type) and thermal resistors (D-type).

Thermal capacitors have thermal capacitance C and elemental equation

$$\boxed{d_t T = \frac{1}{C} q \,.}$$

System components with significant heat capacity (ability to store thermal energy) can be modeled as thermal capacitors.

Thermal resisistors have thermal resistance R and elemental equation

$$\boxed{T = R q \,.}$$

System components that resist heat flow (many) can be modeled as thermal capacitors.

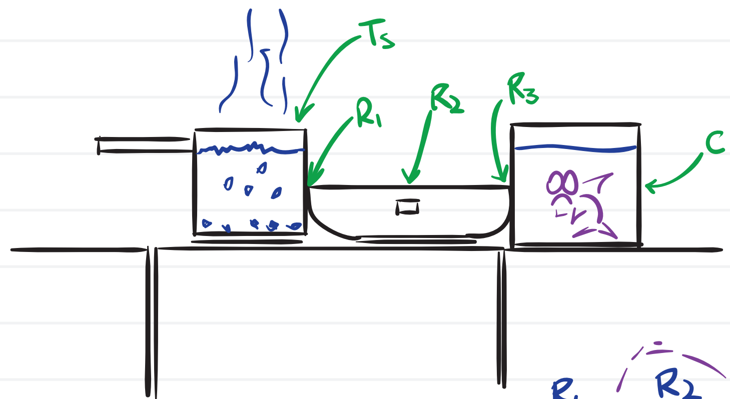
The two types of ideal sources are heat flow rate Q_S and temerature T_S.

Example: fish boil

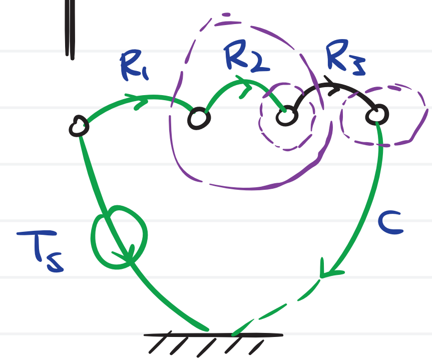
Careless Carlton accidentally left a large pot of water boiling on the stove. Worse, a cast-iron pan is bumped so that it is in solid contact with the pot and his glass fishtank, which is carelessly left next to the stove. Model the sad situation to determine if the fish are gonners.

Solution

1. Schematic.



2. Linear graph + normal tree.



3. Variables + order.

State var's.: T_C State vec.: $\vec{x} = [T_C]$ Order: $n = 1$.

4. Elemental eq's.

C	$d_t T_C = \frac{1}{C} q_C$
R_1	$T_{R_1} = R_1 q_{R_1}$
R_2	$T_{R_2} = R_2 q_{R_2}$
R_3	$T_{R_3} = R_3 q_{R_3}$.

5. Continuity

$$q_C = q_{R_3}$$
$$q_{R_2} = q_{R_3}$$
$$q_{R_1} = q_{R_3}$$

6. Compatibility

$$T_{R_3} = T_S - T_{R_1} - T_{R_2} - T_C$$

7. Algebra (state eq). $d_t T_C = \frac{1}{C} q_{R_3} = \frac{1}{R_3 C} T_{R_3} = \frac{1}{R_3 C}(T_S - T_{R_1} - T_{R_2} - T_C) \Longrightarrow$

$$d_t T_C = \frac{-1}{R_3 C}(T_C + R_1 q_{R_3} + R_2 q_{R_3}) + \frac{1}{R_3 C} T_S = \frac{-1}{R_3 C}(T_C + (R_1 + R_2) C d_t T_C) + \frac{1}{R_3 C} T_S \Longrightarrow$$

$$d_t T_C = \frac{1}{R_3 C\left(1 + \frac{R_1 + R_2}{R_3}\right)}(-T_C + T_S) = \frac{-1}{(R_1 + R_2 + R_3) C} T_C + \frac{1}{(R_1 + R_2 + R_3) C} T_S$$