$\mathcal{L}\{f(t)\} = F(s)$

15.25

$$3\dot{y} + y = 2u$$

We are solving this i/o ODE for a few cases of input + initial conditions.

1. Transfer function.
   a. Take the Laplace transform of the equation.

$$\mathcal{L}\{3\dot{y} + y\} = \mathcal{L}\{2u\}$$

$$3(sY(s) - y(0)) + Y(s) = 2U(s)$$

$$Y(s) = \frac{2U(s) + 3y(0)}{3s+1}$$

$$= \frac{2}{3s+1}U(s) + \frac{3}{3s+1}y(0)$$

$$\Rightarrow \quad H(s) \equiv \frac{Y(s)}{U(s)} = \frac{2}{3s+1} \quad (y(0) \to 0)$$

2. Take the first input + i.c.
   a. Unit ramp $u(t) = t$ and $y(0) = 0$.

$$Y(s) = \frac{2}{3s+1} \cdot \frac{1}{s^2} + \frac{3}{3s+1}(0)$$

$$= \frac{2}{s^2(3s+1)}.$$

   b. Inverse Laplace transform $Y(s) \xrightarrow{\mathcal{L}^{-1}} y(t)$

$$Y(s) = 2\frac{1}{s^2(3s+1)} = \frac{2}{3} \cdot \frac{1}{s^2(s+1/3)} \xrightarrow{\text{table}}$$

$$y(t) = \frac{2}{3} \cdot \frac{1}{(1/3)^2}\left(\frac{1}{3}t - 1 + e^{-t/3}\right)$$

$$= 2t - 6 + 6e^{-t/3} \quad \text{for} \quad t \geq 0$$