An Introduction to Control Theory

Control theory is the study of control systems: systems that control the behavior of other systems. This means that control theory is fundamentally about design as opposed to the analysis of many engineering topics, including system dynamics. In control theory, a system -- usually called the plant -- is analyzed, often with system dynamics, then a control system is designed to control the plant.

A control system usually controls the plant by changing one or more of the plant's inputs. The plant's outputs are variables we would like to know and/or control.

If a plant is mathematically modeled with sufficient accuracy, a controller can specify the plant's input (which is the controller output) to produce the desired output. This is called open-loop control. However, most plant models are not understood well-enough to do this. It is especially difficult to model outside disturbances, like sudden jolts from being bumped and environmental interference.

For this reason, most control systems include feedback: measurements of the plant's outputs. These measurements are "fed back" to the controller, which determines its output (the plant's input) from this information. Determining how the controller should respond to the feedback to control the plant's output is the subject of much of control theory.

There are several types of controllers used in control theory. Which is best for a given system is the primary task of the control engineer. There are design trade-offs to be made. Some controllers will be more expensive to implement than others because they will require more-expensive hardware. Some controllers will perform better than others in some aspects we will now explore.

Transient Response

Transient response of the plant's output is often important for a control system. A designer may have identified such requirements as "the velocity free (initial condition) response must be 0 ± 1 m/s in 5 seconds and thereafter." Or "the pressure step response must not overshoot its final value." This type of requirement is common. As with many design techniques, some iteration is usually needed in order to meet all requirements.

Steady-State Response

Steady-state response of the plant's output is another important consideration for a control system. After the transient response has decayed, the steady-state response must meet certain criteria, such as "the position steady-state response to a unit ramp function must be within 5 mm of the desired position."

Stability

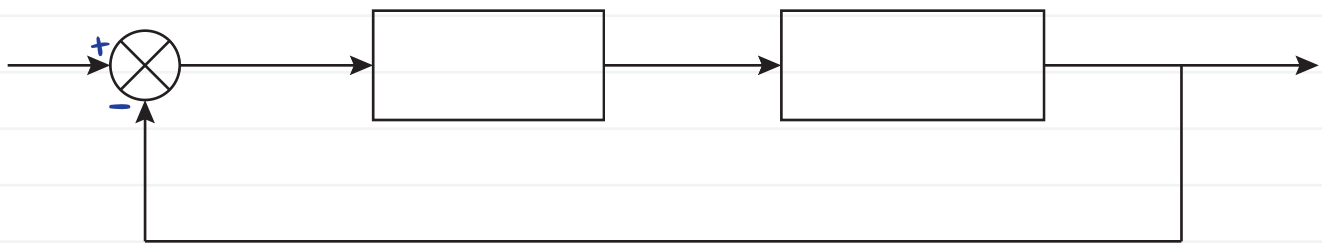
A control system must be designed such that the plant's output response is stable: its free response
must eventually approach zero or it
must eventually approach an oscillation with a constant amplitude.

Others

Cost, weight, complexity, and many other factors must be considered in control system design. One of the most important of these is robustness: the control system's ability to perform as desired when system parameters change from their nominal values. This is important because the parameters of any implementation of a control system will differ from their nominal values at least slightly.

Feedback Control System Block Diagrams

A useful tool for designing control systems is the block diagram. The plant and the controller are represented as blocks. Usually a transfer function (or transfer function matrix) can describe the function of each block. A typical block diagram is shown.



In this configuration, a command function R is provided to the control system. The feedback (plant output) is subtracted from R to give the error E. This is fed to the controller C. The output of the controller is the control effort U, which is the input of the plant G. The output Y is the variable that the control system is attempting to make equal to the command R, therefore, ideally $E = 0$.

Recall that a block diagram can be considered to express algebraic relationships. For instance,

$E(s) = R(s) - Y(s)$, $U(s) = C(s)E(s)$, and $Y(s) = G(s)U(s)$.

Example Solve for the closed loop transfer function $Y(s)/R(s)$.

$Y(s)/R(s)$ can be analyzed to determine if it meets the requirements.