Stability of Control Systems Introduction

Of the three most significant control system specifications -- transient response, steady-state error, and stability -- stability is the most important. We will now turn to stability considerations, limiting ourselves to linear, time-invariant (LTI) systems.

Recall that a system response can be considered to be composed of two parts:
(1) the natural response (or free or initial condition response) and
(2) the forced response.
This terminology will be used throughout the following.

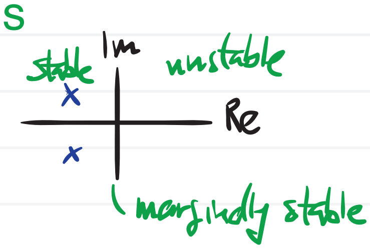
Using the concept of the natural response, we define the following types of stability.
(1) A linear, time-invariant system is stable if the natural response approaches zero as time approaches infinity.
(2) A linear, time-invariant system is unstable if the natural response grows without bound as time approaches infinity.
(3) A linear, time-invariant system is marginally stable if the natural response neither decays nor grows but remains constant or oscillates as time approaches infinity. (Nise)

An alternate formulation of the stability definitions above is called the bounded-input bounded-output (BIBO) definition of stability, and states the following.
(1) A system is stable if every bounded input yields a bounded output.
(2) A system is unstable if any bounded input yields an unbounded output. (Nise)

Marginal stability, then, means that a system has a bounded response to some inputs and an unbounded response to others. For instance, an undamped system response to a sinusoidal input at the natural frequency is unbounded, whereas every other input yields a bounded output.

From our definitions, we see that

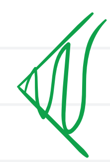
stable systems have closed-loop transfer functions with poles only in the left half-plane. (Nise)



Similarly,

unstable systems have closed- loop transfer functions with at least one pole in the right half-plane and/or poles of multiplicity greater than 1 on the imaginary axis. (Nise)

$\lambda_1 = \lambda_2 \quad y_n = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_2 t}$



Finally,

marginally stable systems have closed-loop transfer functions with only imaginary axis poles of multiplicity 1 and poles in the left half-plane. (Nise)

Identifying stable systems from closed-loop transfer functions

Let the denominator of a c.l. transfer function be the polynomial $b_n s^n + b_{n-1} s^{n-1} + \ldots + b_0 = (s - a_1)(s - a_2) \ldots (s - a_n)$. If a system is stable, it must have all left half-plane poles, so
all a_i must have negative real parts, which implies that
all b_i must be positive and, additionally,
all b_i must be nonzero for $0 \le i \le n$ (i.e. no "missing" powers of s).

However, these b_i conditions are merely necessary conditions for stability, meaning that they are necessary for stability, but not sufficient (something more is needed to ensure stability). However, if they are not met, this is a sufficient condition to draw the conclusion that the control system is unstable (i.e. nothing more needed).