Complex Functions

Complex functions are maps from complex numbers to complex numbers. Recall that the Laplace variable s is a complex variable. Therefore, transfer functions and other Laplace-domain functions are complex functions. We will explore how to evaluate complex functions that are ratios of polynomials, which is the form of a transfer function H(s). Let z_i be zeros and p_j be poles of the transfer function. Then

$$H(s) = \frac{\prod_{i=1}^{n} (s - z_i)}{\prod_{j=1}^{m} (s - p_j)}$$

where n and m are the number of zeros and poles, respectively. Recall that we can write a complex number in polar form with a magnitude and phase. Therefore we can write each of the factors in H(s) as a magnitude and a phase

$$|(s - z_i)| \quad \text{or} \quad |(s - p_j)| \quad \text{and} \quad \angle(s - z_i) \quad \text{or} \quad \angle(s - p_i).$$

Then the magnitude and phase of H(s) are

.

This is an alternative way to compute the value of a transfer funtion for a given value of s. This will be useful when we create a root locus.

There is a graphical interpretation of this method of evaluation of a complex function that will be helpful in the following sections. We will now introduce this through the following figure. (Nise)

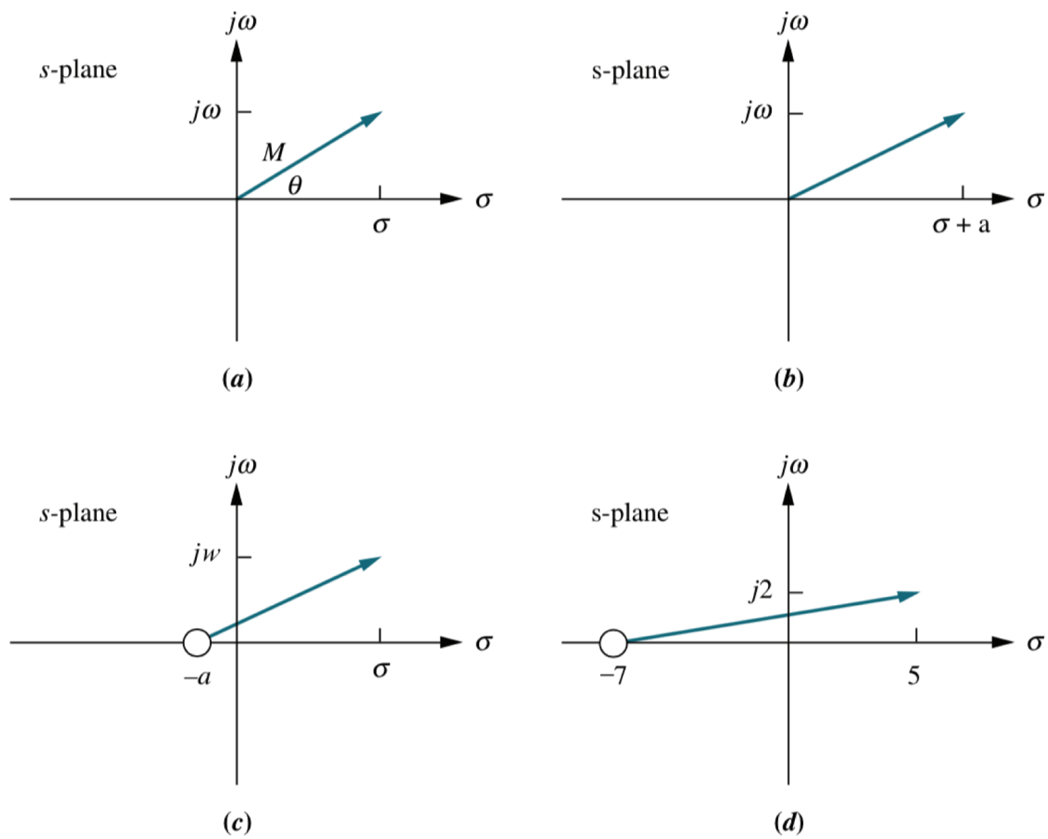

FIGURE 8.2 Vector representation of complex numbers: **a.** $s = \sigma + j\omega$; **b.** $(s + a)$; **c.** alternate representation of $(s + a)$; **d.** $(s + 7)|_{s \to 5+j2}$

In Fig. 8.2a we see a complex number $s = \sigma + j\omega$ represented in the complex plane as a vector with magnitude M and a phase θ. In Fig. 8.2b we have graphed the vector $(s + a) = (\sigma + a) + j\omega$. In Fig. 8.2c, we show an alternative representation of $(s + a)$ that we find useful. Vectors, in some circumstances, can be considered to “float”: they maintain their length and direction, but their absolute placement is undefined. (Technically there are some nuances here with the definition of vector spaces and affine spaces. We’re sweeping that under the rug of “don’t-have-time.”)

This means we can write $(s + a)$ as a vector from the point a to the point s. This has the proper length and angle. Fig. 8.2d has a concrete example.

Example (Nise)

Evaluate the following complex function at $s = -7 + j9$.

$$F(s) = \frac{(s+2)(s+4)}{s(s+3)(s+6)}$$

Solution