Time response of first- and second-order systems: additional characteristics

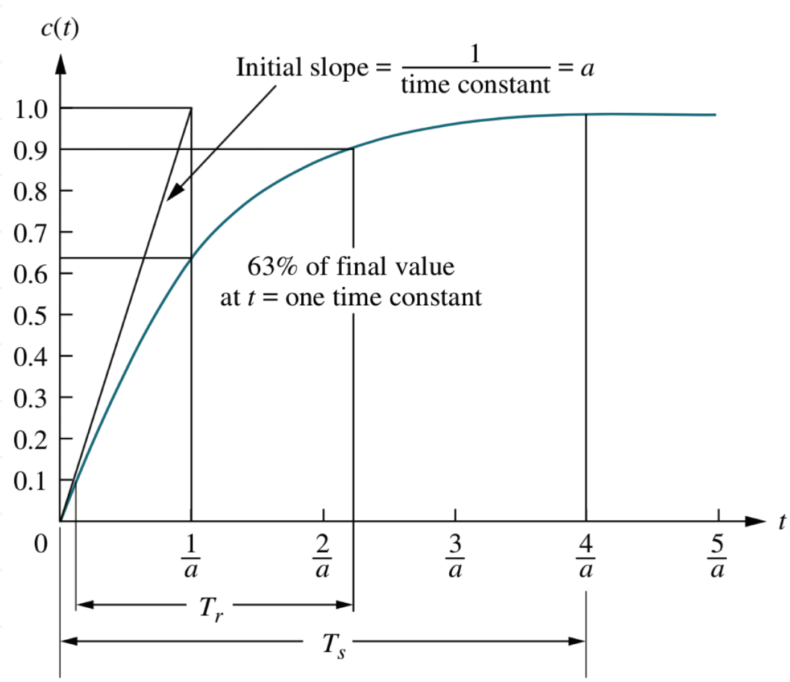
When we examined the time responses of first- and second-order systems, we characterized them by the time constant τ (first-order), natural frequency ω_n (second-order), and the damping ratio ζ (second-order). There are a few other important parameters that characterize the system response of these systems that will be useful in designing control systems.

First-order systems

For first order systems rise time T_r is defined as the time for the waveform to go from 0.1 to 0.9 of its final value (Nise). For first-order systems:

Similarly, the settling time T_s is defined as the time for the response to reach, and stay within, 2% of its final value (Nise). For first-order systems:

The figure illustrates these properties for a unit step response $c(t)$. (Nise)



Second-order systems, underdamped

Second-order, underdamped systems with two poles and no zeros have four additional parameters that are useful to characterize system response.

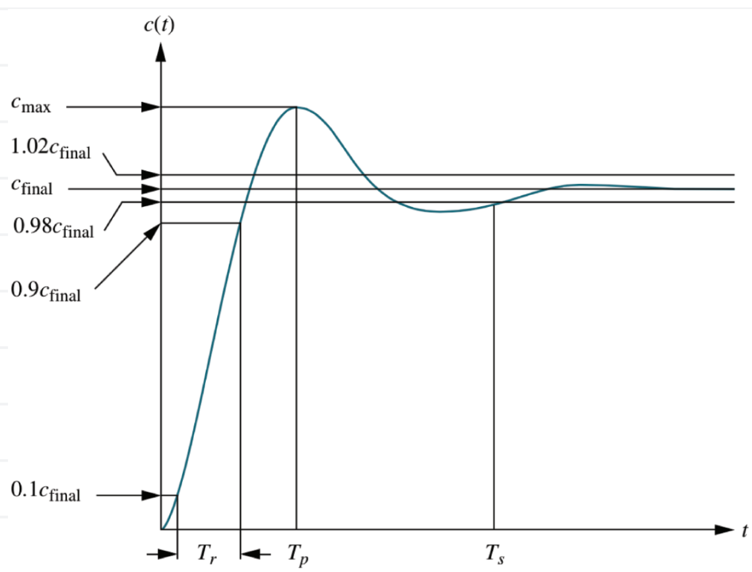
(1) Rise time, T_r. The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value (Nise). There is no closed-form solution for the rise time in terms of ζ and ω_n, but Fig. 4.16 in Nise relates these variables.

(2) Peak time, T_p. The time required to reach the first, or maximum, peak (Nise).

(3) Percent overshoot, %OS. The amount that the waveform overshoots the steady-state, or final, value at the peak time, expressed as a percentage of the steady-state value (Nise).

(4) Settling time, T_s. The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value (Nise).

The figure illustrates these parameters for a unit step response $c(t)$.



Second-order approximation of higher-order systems

Certain higher-order systems can be approximated as second-order systems and can be characterized by the parameters in the preceding section. This includes systems with zeros (in the preceding section we assumed the second-order system had no zeros).

These conditions are as follows (Nise):
(1) Higher-order poles are much farther into the left half of the s-plane than the dominant second-order pair of poles. The response that results from a higher-order pole does not appreciably change the transient response expected from the dominant second-order poles.
(2) Closed-loop zeros near the closed-loop second-order pole pair are nearly canceled by the close proximity of higher-order closed-loop poles.
(3) Closed-loop zeros not canceled by the close proximity of higher-order closed-loop poles are far removed from the closed-loop second-order pole pair.

Higher-order systems controller design proceeds as follows. (Nise)
(1) Sketch the root locus for the given system.
(2) Assume the system is a second-order system without any zeros and then find the gain to meet the transient response specification.
(3) Justify your second-order assumption by finding the location of all higher-order poles and evaluating the fact that they are much farther from the jω-axis than the dominant second-order pair. As a rule of thumb, this textbook assumes a factor of five times farther. Also, verify that closed-loop zeros are approximately canceled by higher-order poles. If closed-loop zeros are not canceled by higher-order closed-loop poles, be sure that the zero is far removed from the dominant second-order pole pair to yield approximately the same response obtained without the finite zero.
(4) If the assumptions cannot be justified, your solution will have to be simulated in order to be sure it meets the transient response specification. It is a good idea to simulate all solutions, anyway.