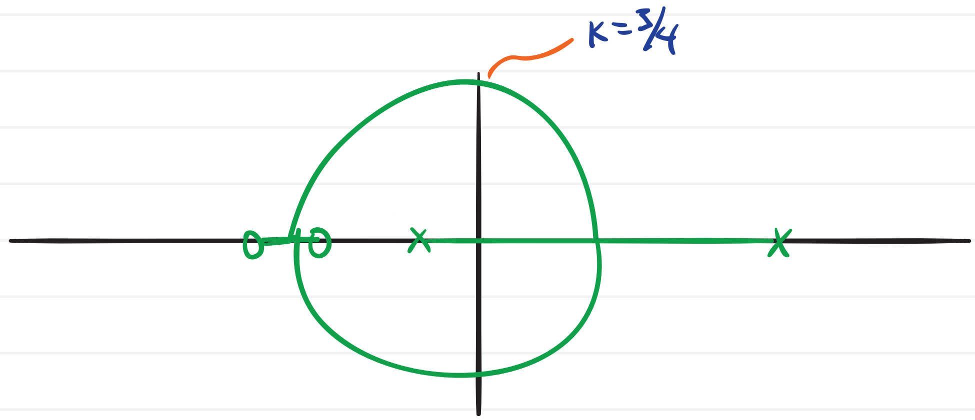
# Example: controlling an unstable system

Given the following open-loop transfer function, design the controller gain $K$ such that the unity negative feedback system has $\%OS = 5$.

$$G(s) = \frac{K(s+3)(s+5)}{(s+1)(s-7)}$$

## Solution

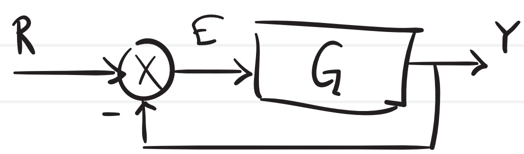
Root locus:




What gain is required for stability?

Routh table:

i. C.L. t.f. :




$$\frac{Y}{R} = \frac{G}{1+G}$$

$$\frac{Y}{R} = \frac{K(s+3)(s+5)}{(s+1)(s-7) + K(s+3)(s+5)}$$

$$= \frac{K(s+3)(s+5)}{s^2 - 6s - 7 + K(s^2 + 8s + 15)}$$

$$= \frac{K(s+3)(s+5)}{(K+1)s^2 + (8K-6)s + (15K-7)}$$

ii. Routh table :

| | | | |
|---|---|---|---|
| $s^2$ | $K+1$ | $15K-7$ | 0 |
| $s^1$ | $\boxed{8K-6}$ | 0 | 0 |
| $s^0$ | $-(15K-7)(8K-6)/(-(8K-6))$ $= 15K-7$ | | |

$K+1 > 0 \Rightarrow K > -1$

$8K-6 > 0 \Rightarrow \boxed{K > 3/4}$

$15K-7 > 0 \Rightarrow K > 7/15$

$\%OS \longrightarrow \zeta$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.69$$

Matlab $\rightarrow$ $K = 2.6$

Compute other params:

$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{2.06} = 1.94 \text{ s} \qquad \omega_n = \frac{2.06}{\zeta} = 2.98 \text{ rad/s}$$

$$T_r = \frac{2.126}{\omega_n} = 0.713 \text{ s}$$