

Lecture 06.02 Difference equations

Many continuous dynamic systems can be described by a linear, constant-coefficient differential equation:

$$\begin{aligned} \alpha_n \frac{d^n y}{dt^n} + \alpha_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + \alpha_1 \frac{dy}{dt} + \alpha_0 y &= \\ = \beta_m \frac{d^m x}{dt^m} + \beta_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \dots + \beta_1 \frac{dx}{dt} + \beta_0 x & \end{aligned} \quad (06.1)$$

where α_k and β_k are constants.

The corresponding discrete system is described by a *difference equation* that operates on the sequence of input values $x(n)$ to produce the output sequence $y(n)$. The difference equation has the form

$$\begin{aligned} a_0 y(n) + a_1 y(n-1) + \dots + a_N y(n-N) &= \\ = b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M) & \end{aligned} \quad (06.2)$$

for $n = 0, 1, 2, \dots$, where $x(n)$ is a sequence of periodically digitized values of the analog input signal, $y(n)$ is a sequence of values that determine the output signal, and a_k for $k = 0, 1, \dots, N$ and b_k for $k = 0, 1, \dots, M$ are constants.

This equation can also be written in summation form:

$$\sum_{k=0}^N a_k y(n-k) = \sum_{k=0}^M b_k x(n-k) \quad (06.3)$$

or, solving this for the current output sample $y(n)$,

$$y(n) = \frac{1}{a_0} \left[\sum_{k=0}^M b_k x(n-k) - \sum_{k=1}^N a_k y(n-k) \right] \quad (06.4)$$

Notice that the current output value $y(n)$ depends on previous values of y and on the previous and current values of the input x .

The problem of finding a discrete approximation of a continuous dynamic system represented by the differential equation [Equation 06.1](#), then, is now just the problem of finding appropriate constants a_k and b_k in the difference equation such that its behavior approximates that of [Equation 06.1](#) with its constants α_k and β_k .

It turns out the best methods of approximation are derived not directly from the differential-difference equation relationship, but instead from the (implied) continuous-discrete transfer function relationship thereof. It is to the discrete transfer function that we therefore turn.