Lecture 03.07 Exact analytical transient response characteristics of first- and second-order systems

03.07.1 First-order systems

First-order systems without zeros have transient responses characterized by a *time-constant* τ that appears in the total response as **time-constant**

$$\text{␣}\, e^{-t/\tau} + \text{␣␣}. \tag{03.34}$$

The transient exponential decays such that in three time constants 3τ only 5 % of the term remains; in 5τ, less than 1 %.

There is neither peak nor overshoot for this type of response. However, the *rise time* for these systems is found from the solution to the first-order input-output ode for $y(0) = 0$: **rise time**

$$y(t) = Kb_0 \left(1 - e^{-t/\tau}\right). \tag{03.35}$$

The rise time is, by definition, the duration of the time interval $[t_1, t_2]$ such that

$$y(t_1) = 0.1 y_{ss} \quad \text{to} \quad y(t_2) = 0.9 y_{ss}. \tag{03.36}$$

The first of these yields

$$\begin{aligned} Kb_0 \left(1 - e^{-t_1/\tau}\right) &= 0.1 Kb_0 \Rightarrow && (03.37a) \\ t_1 &= -\tau \ln 0.9 && (03.37b) \\ &\approx 0.1054\tau. && (03.37c) \end{aligned}$$

Solving in an analogous fashion, we find $t_2 \approx 2.3026\tau$. The interval, then, is $t_2 - t_1 = 2.1972\tau$.

Equation 03.38 first-order system rise time

Finally, the *settling time* can be derived in a fashion similar to the rise time. **settling time**

Equation 03.39 first-order system settling time

03.07.2 Second-order systems

Second-order system transient responses are characterized by a natural frequency ω_n and damping ratio ζ. Following a procedure very similar to that for first-order systems, the following relationships can be derived.

rise time The *rise time* T_r does not have an analytical solution in terms of ω_n and ζ. However, Figure 03.11 can be developed, numerically.

peak time The *peak time* T_p has the following, simple expression

$$T_p = \frac{\pi}{\omega_d}, \tag{03.40}$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency.

percent overshoot The *percent overshoot* %OS is related directly to ζ as follows

$$\%OS = 100 \exp \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \quad \Leftrightarrow \quad \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}. \tag{03.41}$$

settling time Finally, the *settling time* T_s is expressed as

$$T_s = \frac{4}{\zeta\omega_n}. \tag{03.42}$$

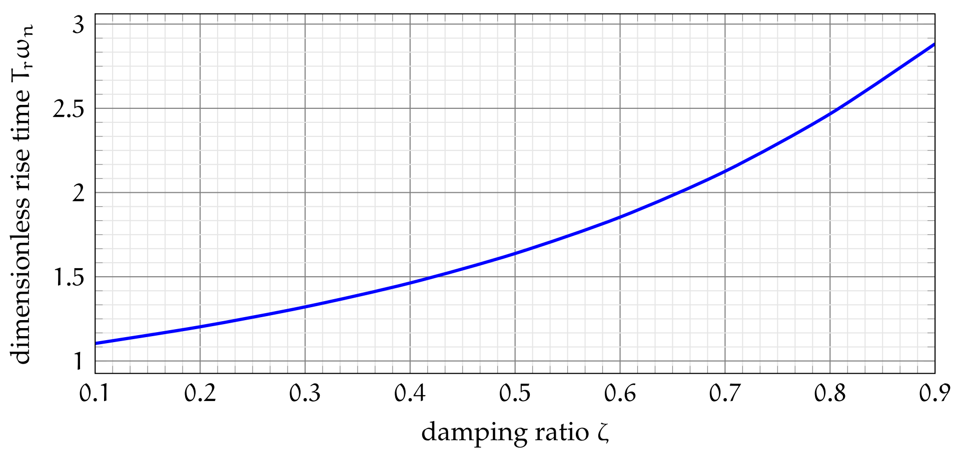

Figure 03.11: the relationship between rise time and damping ratio.