Lecture 03.08 Properties of linear, time-invariant systems

superposition
linear, time-invariant (LTI) systems

From the principle of *superposition*, *linear, time invariant* (LTI) system responses to both initial conditions and nonzero forcing can be obtained by summing the free and forced responses:

$$y(t) = y_{fr}(t) + y_{fo}(t).$$

Moreover, superposition says that any linear combination of inputs yields a corresponding linear combination of outputs. That is, we can find the response of a system to each input, separately, then linearly combine (scale and sum) the results according to the original linear combination. That is, for inputs u_1 and u_2 and constants $a_1, a_2 \in \mathbb{R}$, a forcing function

would yield output

where y_1 and y_2 are the outputs for inputs u_1 and u_2, respectively.

This powerful principle allows us to construct solutions to complex forcing functions by decomposing the problem. It also allows us to make extensive use of existing solutions to common inputs.

There is one more LTI system property worth noting here. Let a system have input u_1 and corresponding output y_1. If the system is then given input $u_2(t) = \dot{u}_1(t)$, the corresponding output is

Similarly, if the same system is then given input $u_3(t) = \int_0^t u_1(\tau) d\tau$, the corresponding output is