Lecture 04.09 Expectation

Recall that a random variable is a function $X : \Omega \to \mathbb{R}$ that maps from the sample space to the reals. Random variables are the arguments of probability mass functions (PMFs) and probability density functions (PDFs).

The *expected value* (or *expectation*) of a random variable is akin to its "average value" and depends on its PMF or PDF. The expected value of a random variable X is denoted $\langle x \rangle$ or $E(X)$. There are two definitions of the expectation, one for a discrete random variable, the other for a continuous random variable. Before we define, them, however, it is useful to predefine the most fundamental property of a random variable, its *mean*.

expected value
expectation
mean

Definition 04.09.1: mean

The *mean* of a random variable X is defined as

$$m_X = E(X).$$

Let's begin with a discrete random variable.

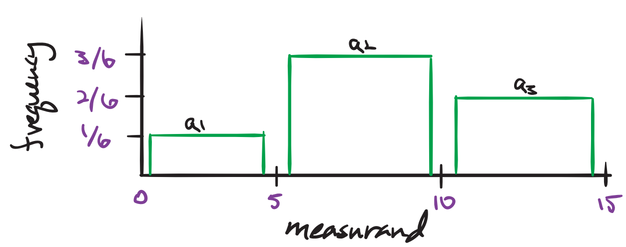
Definition 04.09.2: expectation of a discrete random variable

Let K be a discrete random variable and f its PMF. The *expected value* of K is defined as

$$E(K) = \sum_{\forall k} k f(k).$$

Example 04.09-1 expectation of a discrete random variable

Given a discrete random variable K with PMF shown below, what is its mean μ_K?



Let us now turn to the expectation of a continuous random variable.

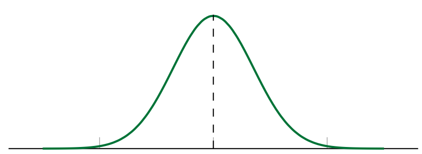
Definition 04.09.3: expectation of a continuous random variable

Let X be a continuous random variable and f its PDF. The *expected value* of X is defined as

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx.$$

Example 04.09-2 expectation of a continuous random variable

Given a continuous random variable X with Gaussian PDF f, what is the expected value of X?



Due to its sum or integral form, the expected value $\mathsf{E}(\cdot)$ has some familiar properties for random variables X and Y and reals a and b.

$$\mathsf{E}(a) = a \tag{04.9a}$$
$$\mathsf{E}(X + a) = \mathsf{E}(X) + a \tag{04.9b}$$
$$\mathsf{E}(aX) = a\mathsf{E}(X) \tag{04.9c}$$
$$\mathsf{E}(\mathsf{E}(X)) = \mathsf{E}(X) \tag{04.9d}$$
$$\mathsf{E}(aX + bY) = a\mathsf{E}(X) + b\mathsf{E}(Y). \tag{04.9e}$$