Lecture A.02 Trigonometry

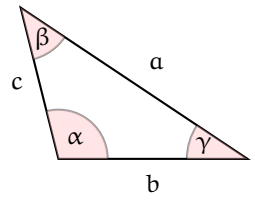
A.02.1 Triangle identities

With reference to the below figure, the *law of sines* is

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \tag{A.3}$$

and the *law of cosines* is

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \tag{A.4a}$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta \tag{A.4b}$$
$$a^2 = c^2 + b^2 - 2cb\cos\alpha \tag{A.4c}$$



A.02.2 Reciprocal identities

$$\csc u = \frac{1}{\sin u} \tag{A.5a}$$
$$\sec u = \frac{1}{\cos u} \tag{A.5b}$$
$$\cot u = \frac{1}{\tan u} \tag{A.5c}$$

A.02.3 Pythagorean identities

$$1 = \sin^2 u + \cos^2 u \tag{A.6a}$$
$$\sec^2 u = 1 + \tan^2 u \tag{A.6b}$$
$$\csc^2 u = 1 + \cot^2 u \tag{A.6c}$$

A.02.4 Co-function identities

$$\sin\left(\frac{\pi}{2}-u\right)=\cos u \tag{A.7a}$$
$$\cos\left(\frac{\pi}{2}-u\right)=\sin u \tag{A.7b}$$
$$\tan\left(\frac{\pi}{2}-u\right)=\cot u \tag{A.7c}$$
$$\csc\left(\frac{\pi}{2}-u\right)=\sec u \tag{A.7d}$$
$$\sec\left(\frac{\pi}{2}-u\right)=\csc u \tag{A.7e}$$
$$\cot\left(\frac{\pi}{2}-u\right)=\tan u \tag{A.7f}$$

A.02.5 Even-odd identities

$$\sin(-u)=-\sin u \tag{A.8a}$$
$$\cos(-u)=\cos u \tag{A.8b}$$
$$\tan(-u)=-\tan u \tag{A.8c}$$

A.02.6 Sum-difference formulas (AM or lock-in)

$$\sin(u\pm v)=\sin u\cos v\pm\cos u\sin v \tag{A.9a}$$
$$\cos(u\pm v)=\cos u\cos v\mp\sin u\sin v \tag{A.9b}$$
$$\tan(u\pm v)=\frac{\tan u\pm\tan v}{1\mp\tan u\tan v} \tag{A.9c}$$

A.02.7 Double angle formulas

$$\sin(2u)=2\sin u\cos u \tag{A.10a}$$
$$\cos(2u)=\cos^2 u-\sin^2 u \tag{A.10b}$$
$$=2\cos^2 u-1 \tag{A.10c}$$
$$=1-2\sin^2 u \tag{A.10d}$$
$$\tan(2u)=\frac{2\tan u}{1-\tan^2 u} \tag{A.10e}$$

A.02.8 Power-reducing or half-angle formulas

$$\sin^2 u = \frac{1-\cos(2u)}{2} \tag{A.11a}$$

$$\cos^2 u = \frac{1+\cos(2u)}{2} \tag{A.11b}$$

$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)} \tag{A.11c}$$

A.02.9 Sum-to-product formulas

$$\sin u + \sin v = 2\sin\frac{u+v}{2}\cos\frac{u-v}{2} \tag{A.12a}$$

$$\sin u - \sin v = 2\cos\frac{u+v}{2}\sin\frac{u-v}{2} \tag{A.12b}$$

$$\cos u + \cos v = 2\cos\frac{u+v}{2}\cos\frac{u-v}{2} \tag{A.12c}$$

$$\cos u - \cos v = -2\sin\frac{u+v}{2}\sin\frac{u-v}{2} \tag{A.12d}$$

A.02.10 Product-to-sum formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right] \tag{A.13a}$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right] \tag{A.13b}$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right] \tag{A.13c}$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right] \tag{A.13d}$$

A.02.11 Two-to-one formulas

$$A\sin u + B\cos u = C\sin(u+\phi) \tag{A.14a}$$

$$= C\cos(u+\psi) \text{ where} \tag{A.14b}$$

$$C = \sqrt{A^2+B^2} \tag{A.14c}$$

$$\phi = \quad \arctan\frac{B}{A} \tag{A.14d}$$

$$\psi = -\arctan\frac{A}{B} \tag{A.14e}$$