Basic Mathematics Reference

1 Quadratic forms

The solution to equations of the form $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{1}$$

1.1 Completing the square

This is accomplished by re-writing the quadratic formula in the form of the left-hand-side (LHS) of this equality, which describes factorization

$$x^2 + 2xh + h^2 = (x + h)^2. \tag{2}$$

2 Trigonometry

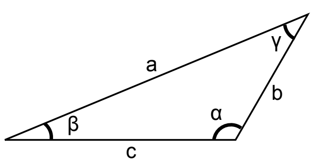
2.1 Triangle identities

With reference to the below figure, the *law of sines* is

$$\frac{\sin\,\alpha}{a} = \frac{\sin\,\beta}{b} = \frac{\sin\,\gamma}{c} \tag{3}$$

and the *law of cosines* is

$$c^2 = a^2 + b^2 - 2ab\,\cos\,\gamma \tag{4a}$$
$$b^2 = a^2 + c^2 - 2ac\,\cos\,\beta \tag{4b}$$
$$a^2 = c^2 + b^2 - 2cb\,\cos\,\alpha \tag{4c}$$



2.2 Reciprocal identities

$$\csc u = \frac{1}{\sin u} \tag{5a}$$
$$\sec u = \frac{1}{\cos u} \tag{5b}$$
$$\cot u = \frac{1}{\tan u} \tag{5c}$$

2.3 Pythagorean identities

$$1 = \sin^2 u + \cos^2 u \tag{6a}$$
$$\sec^2 u = 1 + \tan^2 u \tag{6b}$$
$$\csc^2 u = 1 + \cot^2 u \tag{6c}$$

2.4 Co-function identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \tag{7a}$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \tag{7b}$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \tag{7c}$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \tag{7d}$$
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \tag{7e}$$
$$\cot\left(\frac{\pi}{2} - u\right) = \tan u \tag{7f}$$

2.5 Even-odd identities

$$\sin(-u) = -\sin u \tag{8a}$$
$$\cos(-u) = \cos u \tag{8b}$$
$$\tan(-u) = -\tan u \tag{8c}$$

2.6 Sum-difference formulas (AM or lock-in)

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \tag{9a}$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \tag{9b}$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \tag{9c}$$

2.7 Double angle formulas

$$\sin(2u) = 2 \sin u \cos u \tag{10a}$$
$$\cos(2u) = \cos^2 u - \sin^2 u \tag{10b}$$
$$= 2\cos^2 u - 1 \tag{10c}$$
$$= 1 - 2\sin^2 u \tag{10d}$$
$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u} \tag{10e}$$

2.8 Power-reducing or half-angle formulas

$$\sin^2 u = \frac{1 - \cos(2u)}{2} \tag{11a}$$
$$\cos^2 u = \frac{1 + \cos(2u)}{2} \tag{11b}$$
$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)} \tag{11c}$$

2.9 Sum-to-product formulas

$$\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2} \tag{12a}$$
$$\sin u - \sin v = 2 \cos \frac{u+v}{2} \sin \frac{u-v}{2} \tag{12b}$$
$$\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2} \tag{12c}$$
$$\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2} \tag{12d}$$

2.10 Product-to-sum formulas

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right] \tag{13a}$$
$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right] \tag{13b}$$
$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right] \tag{13c}$$
$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right] \tag{13d}$$

2.11 Two-to-one formulas

$$A \sin u + B \cos u = C \sin(u + \phi) \tag{14a}$$
$$= C \cos(u + \psi) \text{ where} \tag{14b}$$
$$C = \sqrt{A^2 + B^2} \tag{14c}$$
$$\phi = \arctan \frac{B}{A} \tag{14d}$$
$$\psi = -\arctan \frac{A}{B} \tag{14e}$$