Two ways to re-write a complex fraction as a phasor

Way 1.

$$\frac{a+jb}{c+jd} = \frac{c-jd}{c-jd}\cdot\frac{a+jb}{c+jd} = \frac{ca+db+j(cb-da)}{c^2+d^2}$$

$$= \underbrace{\frac{ca+db}{c^2+d^2}}_{\text{def } \sigma} + j\underbrace{\frac{cb-da}{c^2+d^2}}_{\text{def } \omega} = \sigma + j\omega$$

$$= \sqrt{\sigma^2+\omega^2}\, e^{j\phi} \quad \text{where} \quad \phi = \operatorname{atan}\frac{\omega}{\sigma}$$ //

Way 2.

$$\frac{a+jb}{c+jd} = \frac{\sqrt{a^2+b^2}\, e^{j\phi_1}}{\sqrt{c^2+d^2}\, e^{j\phi_2}} \quad \text{where} \quad \phi_1 = \operatorname{atan}\frac{b}{a} + \quad \phi_2 = \operatorname{atan}\frac{d}{c}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}\, e^{j(\phi_1-\phi_2)}$$ //