02.1 stab.intro Introduction

Of the three most significant control system specifications—stability, transient response, and steady-state error—stability is the most important. We will now turn to stability considerations, limiting ourselves to linear, time-invariant (LTI) systems.
Recall that a system response can be considered to be composed of two parts: (1) the free response[1] and (2) the forced[2] response. This terminology will be used throughout the following.

1. The is sometimes called the "natural" or "initial condition" or "unforced" response.
2. The forced response is sometimes called the "input response."

Stability defined by the free response

Using the concept of the free response, we define the following types of stability for LTI systems[3].

3. N. Nise, 2015.

1. An LTI system is **assymptotically stable** if the free response approaches zero as time approaches infinity.
2. An LTI system is **unstable** if the free response grows without bound as time approaches infinity.
3. An LTI system is **marginally stable** if the free response neither decays nor grows but remains constant or oscillates as time approaches infinity.

Stability defined by the forced response

An alternate formulation of the stability definitions above is called the **bounded-input bounded-output** (BIBO) definition of stability, and states the following[4].

4. **ibidem**.

1. A system is **BIBO stable** if every bounded input yields a bounded output.
2. A system is **BIBO unstable** if any bounded input yields an unbounded output.

In terms of BIBO stability, **marginal stability**, then, means that a system has a bounded response to some inputs and an unbounded response to others. For instance, a second-order undamped system response to a sinusoidal input at the natural frequency is unbounded, whereas every other input yields a bounded output.

Although we focus on the definitions of stability in terms of the free response, it is good to understand BIBO stability, as well.