

06.10 rldesign.multd Multiple derivative compensators

Note: this section is exploratory and incomplete.

Lec. 06.6 rldesign.PD shows how to design a derivative compensator such that the compensated root locus of a control system can be made to include some test point $\psi \in \mathbb{C}$ where the designer would like a closed-loop pole (typically to satisfy transient response requirements). This derivative compensator has the form

$$C_D = K(s - z_c), \quad (1)$$

for gain $K \in \mathbb{R}$ and zero $z_c \in \mathbb{R}$. The crux of the design procedure is to compute via the root locus phase criterion¹¹ the *required* compensator phase contribution:

$$\theta_c = \pi - \angle GH(\psi) \quad (2)$$

for open-loop transfer function $GH(s)$. A trigonometric analysis shows that, for $\theta_c \in [-\pi, \pi]$, the compensator zero must be

$$z_c = \operatorname{Re}(\psi) - \operatorname{Im}(\psi) / \tan \theta_c. \quad (3)$$

The obvious limitation here is that if the required compensation θ_c is beyond $\pm\pi$, the derivative compensator of Eq. 1 cannot contribute sufficient phase. The strategy we adopt here is to augment the derivative compensator to include as many (equal) zeros as we need:

$$C_m = K(s - z_m)^m, \quad (4)$$

where z_m is a zero of multiplicity m . We call this a **multiple derivative compensator** or *m-derivative compensator*.

¹¹The phase criterion was defined in Lec. 05.1 rlocus.def, Eq. 6.

Algorithm multd.1 the multiple derivative compensator algorithm.

```

function d_comp_m( $\psi$ , GH( $s$ ))
   $\theta_c \leftarrow \pi - \angle \text{GH}(\psi)$  ▷ required phase comp
   $m \leftarrow \text{ceiling}(\theta_c/\pi)$  ▷ zeros needed
   $\theta_m \leftarrow \theta_c/m$  ▷ divide contributions
   $z_m \leftarrow \text{Re}(\psi) - \text{Im}(\psi)/\tan \theta_m$  ▷ trig
   $C'_m \leftarrow (s - z_m)^m$  ▷ comp sans gain
   $K_m \leftarrow |C'_m(\psi)\text{GH}(\psi)|^{-1}$  ▷ angle criterion
   $C_m \leftarrow K_m C'_m$  ▷ comp with gain
  return  $C_m$ 
end function

```

How do we select the compensator zero z_m and multiplicity m for a given θ_c ? First, we determine m by determining how many π (or $-\pi$) contributions are required:^{12,13}

$$m = \left\lceil \frac{|\theta_c|}{\pi} \right\rceil. \quad (5)$$

With this, we can divide-up the the required phase contribution θ_c among the m zeros:

$$\theta_m = \theta_c/m. \quad (6)$$

By construction, $\theta_m \in [-\pi, \pi]$, so the compensator zeros should be located at

$$z_m = \text{Re}(\psi) - \text{Im}(\psi)/\tan \theta_m. \quad (7)$$

This is summarized in [Algorithm multd.1](#).

Causality

A complication can arise when derivative compensation yields a closed-loop transfer function with more zeros than poles—a type of system called **non-causal** (non-non-causal systems are called **causal**). Non-causal

¹²The function $\lceil \cdot \rceil$ is called the ceiling function and rounds up to the nearest integer.

¹³Note that if $\theta_c \in [-\pi, \pi]$, the multiplicity $m = 1$ and the compensator is a regular derivative compensator.

systems are those that depend on *future* states, something classically¹⁴ impossible to instantiate in real-time, and therefore a controller that creates such a control system is of no practical use.¹⁵ Adding multiple zeros to a controller can easily yield such undesirable systems.

Algorithm multd.2 the multiple derivative compensator algorithm with ι integrators.

```

function d_comp_mi( $\psi$ , GH( $s$ ),  $\iota$ )
     $\theta_c \leftarrow \pi - \angle \text{GH}(\psi)/s^\iota$            ▷ required phase comp
     $m \leftarrow \text{ceiling}(\theta_c/\pi)$              ▷ zeros needed
     $\theta_m \leftarrow \theta_c/m$                  ▷ divide contributions
     $z_m \leftarrow \text{Re}(\psi) - \text{Im}(\psi)/\tan \theta_m$            ▷ trig
     $C'_m \leftarrow (s - z_m)^m/s^\iota$            ▷ comp sans gain
     $K_m \leftarrow |C'_m(\psi)\text{GH}(\psi)|^{-1}$        ▷ angle criterion
     $C_m \leftarrow K_m C'_m$                    ▷ comp with gain
    return  $C_m$ 
end function

```

To mitigate this, we can include ι pure integrators $1/s$ into the compensator. They will obviously affect the root locus, so their effects must be taken into account during the zero compensator calculations. This is done by treating the open-loop transfer function as if it already had the compensator integrators $1/s^\iota$. **Algorithm multd.2** summarizes this approach.

Example 06.10 rldesign.multd-1

Design a controller to meet the ...

¹⁴It gets complicated when considering relativity and quantum mechanics, which we do not, here.

¹⁵Non-causal system models are useful for digital signal post-processing, but these are always *a posteriori*—i.e. “future” time is known because it is in the analytic past. Controllers do not have this luxury.