07.1 freq.intro Introduction

The **frequency response function** $H(j\omega)$ is a complex function that relates a system's input u to its output y in terms of the input's frequency content. Given a transfer function $H(s)$ (which also relates u to y), the frequency response function can be found by the substitution

$$H(j\omega) = H(s)|_{s \to j\omega} . \tag{1}$$

It can be shown that, for a system with input $u(t) = A \sin(\omega t + \psi)$, with $A, \omega, \psi \in \mathbb{R}$ being the amplitude, angular frequency, and phase of the input, and frequency response function $H(j\omega)$, the steady-state output is

Equation 2 frequency-dependent sinusoidal response

where $|H(j\omega)|$ and $\angle H(j\omega)$ are the magnitude (i.e. norm) and phase of $H(j\omega)$, respectively. There are three striking aspects of this equation:

1. the output is also a sinusoid at the same frequency as the input;
2. the output amplitude is the input amplitude scaled by $|H(j\omega)|$; and
3. the output phase is the input phase plus $\angle H(j\omega)$.

With Fourier Series and Fourier Transform representations of signals, we can consider the input to be composed of sinusoids. For LTI systems, the principle of superposition allows us to construct a corresponding output representation.

In Lec. 07.2 freq.bode and Lec. 07.5 freq.nyquist, we introduce the two primary ways $H(j\omega)$ is plotted. Lec. 07.6 freq.nystab explores what we can learn about system stability from $H(j\omega)$ and its plots. Finally, we learn how the different time-domain and frequency-domain representations of a system are related Lec. 07.8 freq.freqtime.

The frequency response methods of this chapter were actually developed before root locus methods, and are equivalent in many ways. We learn these methods for two reasons: first, they give us a deeper understanding of control systems and second there are a few situations for which frequency response methods are preferred:

1. when constructing a transfer function from measurement data,
2. when designing a controller for transient and steady-state response characteristics with lead compensation (sans lag compensation), and
3. when determining the stability of a nonlinear system.