stab.routh Routh-Hurwitz criterion

There is no practical way to find the roots of a polynomial greater than degree four.⁷ An implication of this is that we cannot practically solve (analytically) for the poles of a closed-loop transfer function with degree greater than four. Fortunately, **numerical root finders** can handle these higher-order systems with ease. However, there is a drawback to us<u>ing</u> numerical root finders to determine stability: design parameters, which show up in the coefficients of the denominator polynomial of a transfer function, must be assigned a specific value.

A couple of mathematicians⁸ in the late 19th century came up with a clever test-called the **Routh-Hurwitz stability criterion**⁹-for learn<u>ing</u> much about the stability of a system without comput<u>ing</u> its poles; moreover, the test yields an analytically tractable way to determine ranges over which design parameters yield stable closed-loop systems.

An algorithm for applying the Routh-Hurwitz criterion

We consider an algorithm for this test. First, we address the "basic" algorithm and refer the reader to Nise¹⁰ for the two exceptions that arise when Column 1 has a zero or when an entire row is zero. You can teach this algorithm (includ<u>ing</u> the exceptions) to a computer, as some have, but it is easy enough by-hand for many systems. 7. For the interested reader, see this stackexchange discussion.

numerical root finders

8. Edward John Routh and Adolf Hurwitz were their names.

Routh-Hurwitz stability criterion

9. It is noteworthy that the criterion is based on the Routh-Hurwitz theorem.

10. Nise, Control Systems Engineering, 7th Edition.

Let the denominator of a closed-loop transfer

b₁

C1

d1

91

 $a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n$

where n a finite integer greater than or equal to the order of the numerator polynomial and $a_0 > 0$ (if it is not, make it so by multiplication by -1). Perform the following two steps.

First, constr is to fill in t shown in Ta

Table routh.1: the general form of the Routh table. Empty cells are always zero.

$$\begin{aligned} 1 & 2 & 3 & 4 & \cdots \\ 1 & a_{0} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{0} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{2} & a_{4} & a_{6} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{3} & a_{5} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{6} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{6} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{6} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{7} & a_{7} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{8} & a_{7} & a_{1} & a_{1} & a_{7} & a_{7} & \cdots & \cdots \\ a_{1} & a_{1} & a_{1} & a_{7} & a_{7} & \cdots$$

Note the pattern that emerges in Equation 1. The number of rows and potentially nonzero columns are n + 1 and [(n + 1)/2]. Potentially nonzero values hug Column 1. Descending rows, the number of potentially nonzero coefficients decreases.

The second step is to interpret the Routh table. basic Routh table interpretation For the basic Routh table, no poles lie on the

imaginary axis (which excludes marginal stability), so interpretation is simple: the number of sign changes in Column 1 is equal to the number of poles in the right half-plane-and all others are in the left half-plane. Therefore, the system is strictly stable if its Routh array is of the basic type and has no sign changes in Column 1.

Example stab.routh-1

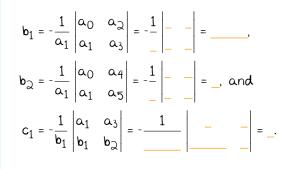
Given the closed-loop transfer function

$$\frac{s+7}{s^3+3s^2+s+K}$$
 (2)

where K is a design parameter, using the Routh-Hurwitz criterion, find the range of K for which the closed-loop system is stable.

Let's build the Routh table in Table routh.2.

The lower entries were computed from Equation 1 (n.b. we knew $b_2 = 0$, but compute it for demonstrative purposes) as follows:

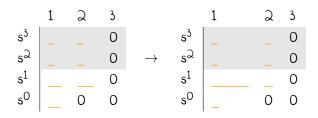


Now we must interpret the result. Since the first two entries in Column 1 are positive, the last two must be in order for the system stability. The conditions are:

$$\underline{} > 0 \Rightarrow \underline{}$$
 and
 $K > \underline{}$.

re: Basic Routh table with an unknown parameter

Table routh. 2: Routh table for Example stab.routh-1.



Therefore, the range for stability is _____. Expressed as an interval, $K \in ___$.