rldesign.gain   Gain from the root locus

In what follows, we will be primarily concerned with locations on the root locus that yield desirable transient response characteristics. But knowing the location on the root locus at which we would like to operate requires that we know the implicit gain associated with placing the closed-loop poles at that location. This lecture demonstrates two methods of finding the gain associated with a given location on the root locus.

Gain, analytically and geometrically

Recall that in Lec. rlocus.def, we defined the root locus magnitude criterion to be

(1)

Solving this for K, we obtain the following result, which is valid for a test point $s = \psi$ on the root locus.

> **Equation 2   gain from the magnitude criterion**
>

Consider this in terms of the gain k, poles p_j, and zeros z_i of G(s)H(s) (note that k is exclusive of the controller gain K). Eq. 2 can be written as a function of a test point ψ on the root locus. (See Appendix A.01 for details on this representation.)

> **Equation 3   geometric gain**
>

From the discussion of Appendix A.01, we obtain the geometric interpretation: the gain is equal

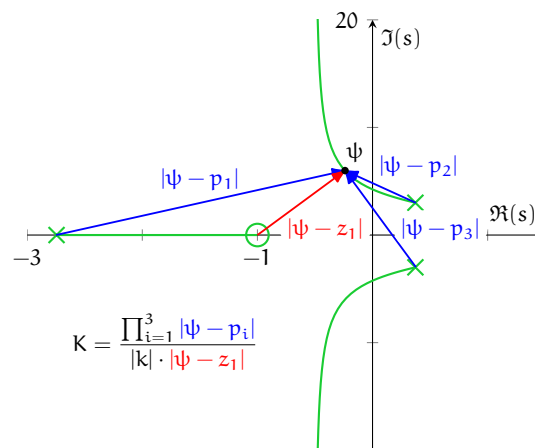$$K = \frac{\prod_{i=1}^{3} |\psi - p_i|}{|k| \cdot |\psi - z_1|}$$

Figure gain.1: an example of a geometric interpretation of the the loop gain K for a closed-loop system with transfer function poles $p_{1,2,3}$, zero z_1, and gain K at a complex value $s = \psi$ on the root locus.

to the the product of the lengths of vectors
that originate at poles and terminate at the
test point divided by the product of K with
the lengths of vectors that originate at the
zeros and terminate at the test point. This is
illustrated in Fig. gain.1.

Gain, the easy way

Finding the loop gain K that yields a
closed-loop pole at a specific point on the
root locus is much easier to find numerically.
Recall that for a given value of gain K, the
closed-loop poles are easily found numerically.
Software typically generates root locus plots
in a brute-force way: by simply computing the
closed-loop pole locations for a range of gains.
Therefore, the gain corresponding to each point
on these plots is then known **a priori**.

In programs such as MATLAB, the data cursor
will typically display the gain corresponding
to each point. If insufficient resolution is
available, more can typically be specified with
an optional argument, as in MATLAB's rlocus as
follows.

```
G = zpk([-10,-20],[-5,5,-3],1); % transfer function
k = [logspace(-1,4,1e4),Inf]; % custom gains
figure;
rlocus(G,k) % plot with custom gains
```