freq.nybode Stability, GM, and PM from Bode plots

Bode plots are an alternative representation of the positive $j\omega$-axis Nyquist plot. As we established in Lec. freq.nystab, this is sufficient information for stability via the Nyquist criterion. In a similar fashion, the gain and phase margins can also be found from the positive $j\omega$-axis Nyquist plot.

For these reasons, the stability, gain margin, and phase margin can all be found from the Bode plot. In fact, this is the preferred method for finding the gain and phase margins.

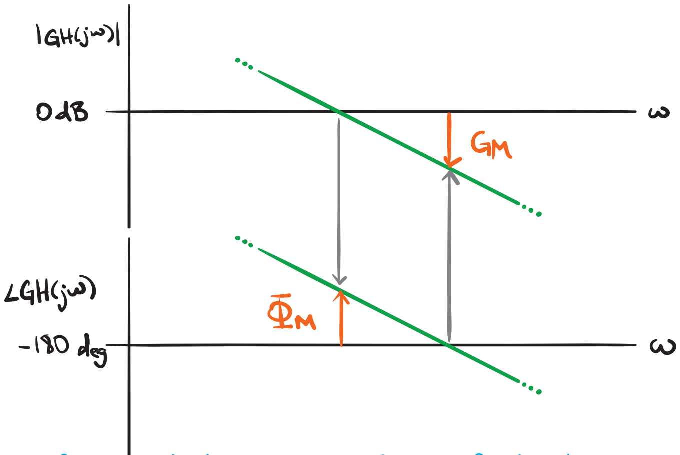
It is common, but somewhat risky, practice to simply use the Bode plot to determine stability, gain margin, and phase margin. Here is why it is risky: when we use it, we assume that the system

1. is open-loop stable;
2. with sufficient gain, has only clockwise encirclements of -1; and
3. has a single negative-real-axis crossing.

Although these are all commonly met, there are plenty of systems for which they are not. For this reason, we encourage caution with this common practice. We proceed by describing the method.

Recall that the gain margin G_M is defined by the distance between the negative-real-axis intercept of a Nyquist plot and -1. This occurs at -180 deg. On a Bode plot, such as that of Fig. nybode.1, it is easy to determine the dB magnitude difference from the magnitude at -180 deg and 1 = 0 dB.

Similarly, recall that the phase margin Φ_M is defined as the difference between the angle at magnitude 1 = 0 dB and -180 deg as the Nyquist plot approaches -180 deg. On a Bode plot, such as that of Fig. nybode.1, the magnitude 0 dB point near -180 deg corresponds to the phase from which Φ_M can be determined.



re: Gain and phase margin from a Bode plot

Figure nybode.1:

Example freq.nybode-1

Let the open-loop transfer function GH(s) be defined as

$$GH(s) = \frac{1}{s^3 + 2s^2 + 5s + 6}.$$

Determine closed-loop stability, gain margin, and phase margin from an open-loop Bode plot.