

## Voltage, current, + resistance (adapted from AOE, ch1)

Two quantities are of special importance in electrical circuits: voltage and current. The relationship between them defines a third important quantity: resistance (or impedance).

We begin by defining each of these.

### Voltage

Definition: **Voltage** is the **difference** in electrical potential energy of a unit of **charge** moved between two locations in an electrical field.

It is typically denoted  $v$ , and has units **volts** with symbol  $V$ .

Voltage is always defined by referring to **two** locations. Sometimes this location is implicitly the **ground**, which is a reference for a circuit, typically defined to have zero electrical potential energy.

### Current

Definition: **Current** is a flow of charge.

It is typically denoted  $i$ , and has units **amperes** with symbol  $A$ .

#### Some terminology

We **generate** voltages by doing work on charges with devices like batteries (electrochemical work), generators (magnetic work), and solar cells (photovoltaic conversion).

We **get** currents by placing voltages across elements, through which current flows.

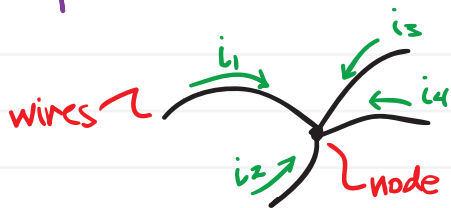
# Circuits

**Circuits** are dynamic electrical systems in which charge accumulates and flows through **energy storage elements** (e.g. capacitor). Energy storage elements are connected via metallic **conductors** called wires, which usually (ideally) have the same voltage (relative to, say, ground) everywhere.

## Kirchoff's Current Law

Charge is a fundamental physical property that is **conserved**. This means that, given any point in a circuit that isn't accumulating charge (e.g. wires), the currents flowing **in** must equal those flowing **out**.


### Example



Kirchoff's current law implies that

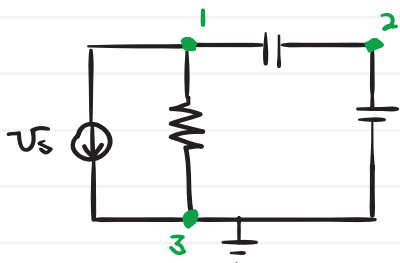
$$i_1 + i_2 + i_3 + i_4 = 0 \quad \text{or} \quad \sum_{j=1}^4 i_j = 0.$$

## Kirchoff's Voltage Law

Circuit elements connected in **parallel**, like , have the same voltage across them.

Note that this doesn't imply that the current through the elements is the same. However, it does imply that **the sum of the voltage drops around any closed loop in a circuit is zero**.

### Example



Kirchoff's voltage law implies

$$V_{12} + V_{23} + V_{31} = 0 \quad \text{and}$$

$$V_5 + V_{31} = 0$$

# Power

The **instantaneous power** being consumed by a circuit element at a given moment is

$$P(t) = v(t)i(t)$$

instantaneous  
voltage across  
the element

instantaneous  
current through  
the element

Power typically goes into:

- heat (usually),
- mechanical work (motors),
- radiated energy (lamps, transmitters), or
- stored energy (batteries, capacitors).

## More on terminology

"[D]on't call current 'amperage'; that's strictly bush-league.  
The same caution will apply to the term 'ohmage'...."

— Horowitz + Hill, The Art of Electronics

# Resistors

Roughly: electronics is all about making and using circuit elements with interesting  $i-v$  relationships.

Examples:

- resistors ( $i$  proportional to  $v$ ),
- capacitors ( $i$  proportional to rate-of-change of  $v$ ),
- diodes ( $i$  flows in only one direction),
- thermistors (temperature-dependent resistors),
- photoresistors (light-dependent resistors), and
- strain gauges (strain-dependent resistors).

Metal conductors with high conductivity tend to quickly transfer around any concentrations of charge, thus leaving very little potential difference (voltage). We use these as **wires**.

However some metal conductors with low conductivity have the interesting property that the voltages across them are proportional to the currents through them.


A **resistor** is made out of these types of conductors (e.g. carbon or thin metal) with a wire coming out of each end. It is characterized by its **resistance**:

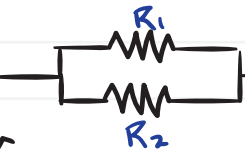
$$R \equiv v/i, \leftarrow \text{Ohm's Law}$$

which is often assumed to be constant for certain operating regimes (e.g. temperature) of a given resistor.

$R$  has SI units of Ohms ( $\Omega$ ).

Roughly speaking, resistors are used to convert voltage to current and current to voltage.

Resistors in **series**  can be considered to be a single resistor with resistance  $R_{eq} = R_1 + R_2$ .

Resistors in **parallel**  can be considered to be a single resistor with resistance

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2}$$