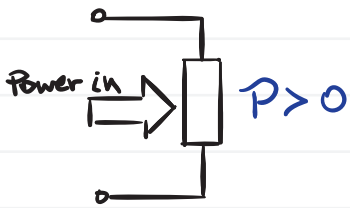
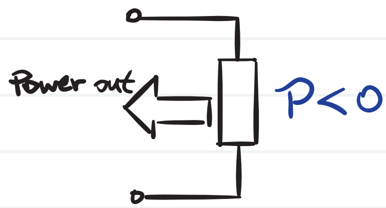
Circuit analysis

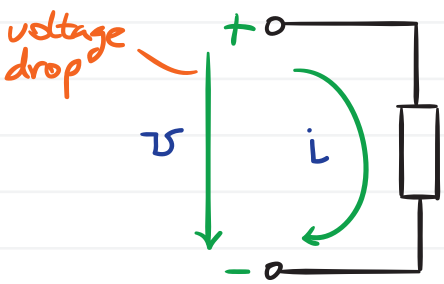
Sign convention

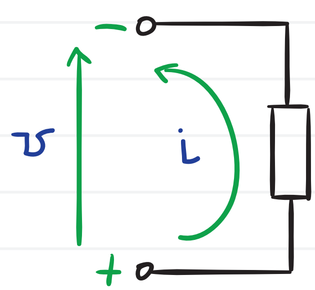
The passive sign convention of electrical engineering is used. We consider power into a component to be positive and power flowing from a component to be negative.



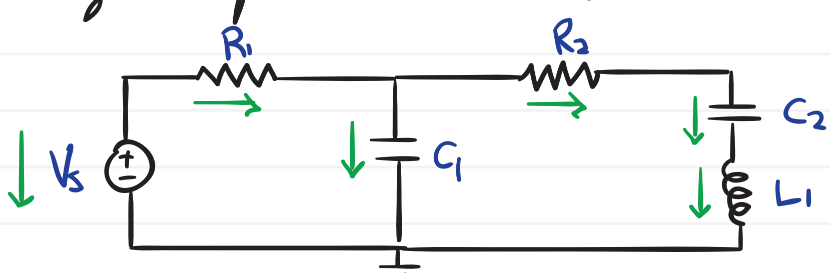


Because $P = vi$, this implies that the current and voltage signs are prescribed by the convention. The electrical potential must drop in the direction of positive current flow. This means the assumed direction of voltage drop across an element must be the same as that of the current flow.





When analyzing a circuit, for each element, draw an arrow beside it pointing in the direction of assumed current flow and voltage drop. For instance,



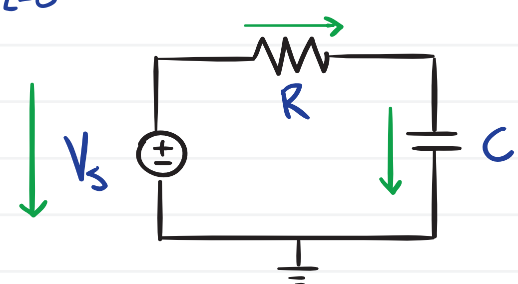
Analysis of a circuit

After prelinaries (e.g. diagram, sign convention), analysis of a circuit begins with the writing of the elemental equations for all the elements.

The next step is to write the continuity and compatibility equations of Kirchhoff. Around each loop, apply Kirchhoff's voltage law. For each node, apply Kirchhoff's current law.

The resulting system of algebraic and differential equations can be solved for given inputs and initial conditions.

Example Given the RC-circuit below with $V_s = 12V$ after $t=0$ and $v_c(t)\big|_{t=0} = 0$, what is $v_c(t)$?



1. Elemental eq's: $v_R = i_R R$ ① & $i_c = C\frac{dv}{dt}$. ②

2. KVL: $v_R + v_c - V_s = 0$. ③
   KCL: $i_R - i_c = 0$. ④

3. With this simple system, we can easily find the ODE we must solve. From ④, $i_R = i_c = i$. From ③, ①, & ②,

$$v_R + v_c - V_s = 0$$
$$iR + v_c = V_s$$
$$RC\frac{dv_c}{dt} + v_c = V_s \quad ⑤$$

All that remains is to solve ⑤ with the initial cond.

Homogeneous sol'n:

The characteristic eq. is $\tau\lambda + 1 = 0$ (let $\tau \equiv RC$)

and

$$\lambda = -1/\tau .$$

The solution is $v_{c_h}(t) = K_1 e^{\lambda t} = K_1 e^{-t/\tau}$.

Particular sol'n:

From the form of the input, $v_{c_p}(t) = K_2$.

Plugging into 5,

$$\tau(0) + K_2 = V_0 \qquad (V_0 = 12\,V)$$

$$\Rightarrow \quad K_2 = V_0 .$$

Total sol'n: $v_c(t) = v_{c_h}(t) + v_{c_p}(t) = K_1 e^{-t/\tau} + V_0$

Specific sol'n: apply initial conditions: $0 = K_1 e^{-0/\tau} + V_0$

$\Rightarrow K_1 = -V_0$.

Therefore, $v_c(t) = (V_0 - V_0 e^{-t/\tau}) = V_0(1 - e^{-t/\tau})$.