DC motor and battery

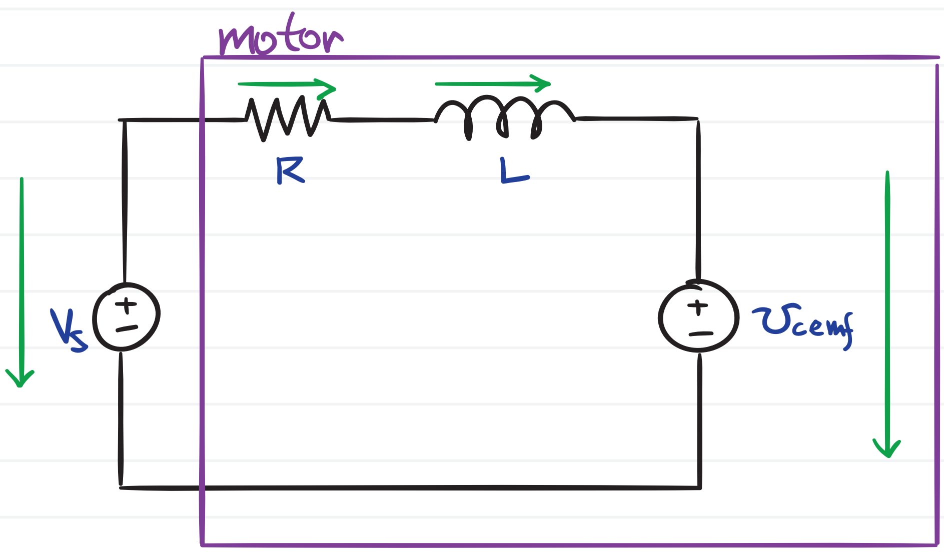
Two common DC circuit elements are batteries and DC motors. We will model each.

DC motors

DC motors exert a torqe $T = K_t i$ (where K_t is the torque motor constant and i is the motor current) on a rotational mechanical load.

A motor's armature is typically a wire coil through which current flows. A magnetic field is applied such that the coil rotates due to its magnetic field interaction (called the Lorentz force).

This can be modeled by the following circuit.



The resistor R and inductor L account for the armature's resistance + inductance.

The mysterious voltage source v_{cemf} is called the counter-emf or back-emf. As the motor speeds up, Faraday's Law predicts a voltage will be generated in opposition to the original (no velocity) voltage. This behavior is that which makes a motor behave as a generator.

The back-emf is usually characterized by a constant of proportionality K_v which is used to relate motor speed + back-emf.

$$v_{emf} = K_v \Omega .$$

The motor constants K_t + K_v are usually provided by the manufacturer of the motor.

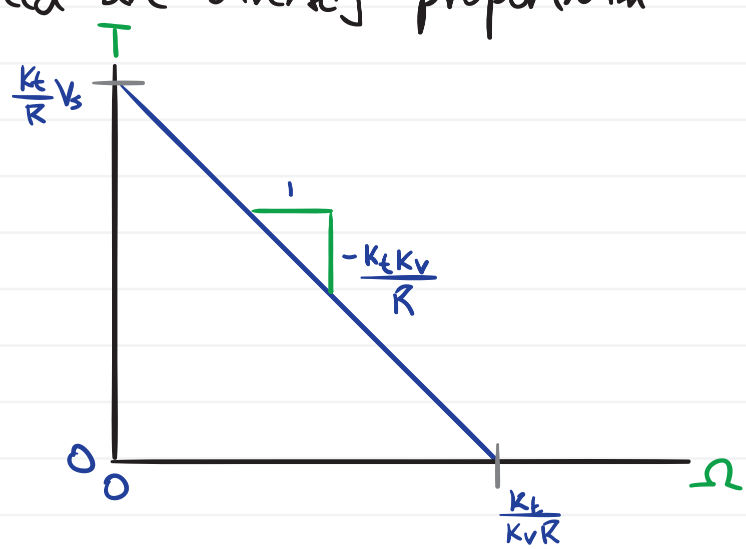
Assuming steady-state operation (i.e. DC current applied), we can ignore the inductance L. The current i is of primary importance to us so we compute it:

$$i = \frac{v_R}{R} = \frac{1}{R}(V_s - v_{emf}) = \frac{1}{R}(V_s - K_v \Omega) .$$

So the current drops as the load speeds up. This has a direct effect on the torque:

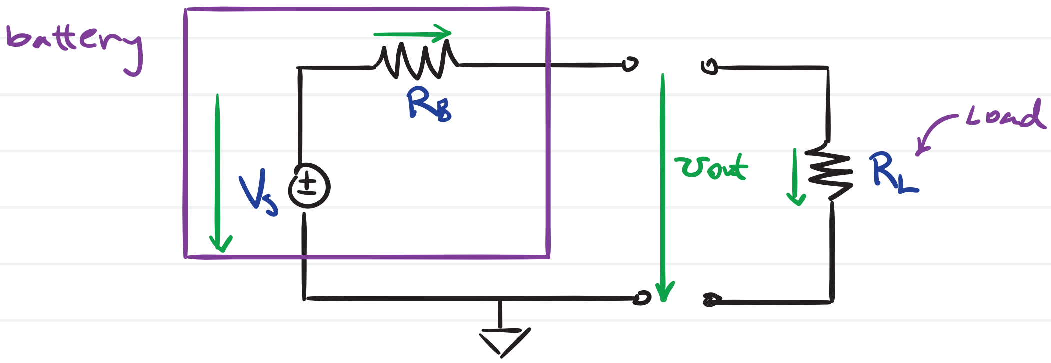
$$T = K_t i = \frac{K_t}{R}(V_s - K_v \Omega) .$$

This relationship predicts the familiar result that motor torque and speed are inversely proportional



Batteries

A battery can be modeled as an ideal voltage source in series with a resistor.



The battery's internal resistance yields the output voltages:

no load : $v_{out} = V_s - v_{R_B} = V_s - \overset{0}{\cancel{i}} R_B = V_s$

load : $v_{out} = V_s - v_{R_B} = V_s - iR_B = V_s - \frac{v_{out}}{R_L} R_B$

$$\Longrightarrow \quad v_{out} = \frac{R_L}{R_L + R_B} V_s \; .$$