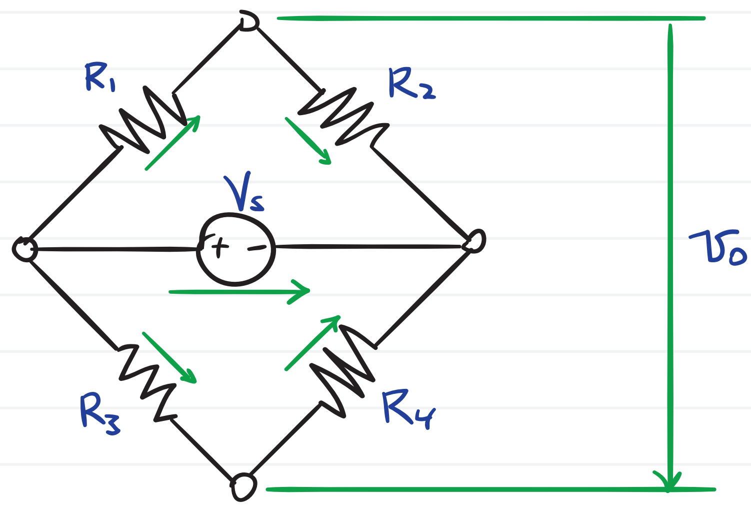
Wheatstone bridges

The Wheatstone bridge circuit is one of the most important for measurements. We will discuss a number of applications for Wheatstone bridges, including strain gages, pressure transducers, and hot-wire anemometers.



Analysis

1. Assign sign conventions.

2. Elemental equations $v_{R_j} = R_j i_j$ (4 of them)

3. KCL: $i_{R_1} = i_{R_2}$ $i_{R_3} = i_{R_4}$

4. KVL: $V_s = v_{R_3} + v_{R_4}$ $V_s = v_{R_1} + v_{R_2}$

5. Solve for $v_o(V_s)$. v_o is $v_o = v_{R_3} - v_{R_1} = R_3 i_3 - R_1 i_1$.
From KVL + KCL,

$V_s = i_3 R_3 + i_4 R_4 \Rightarrow i_3 = \frac{1}{R_3+R_4} V_s$ | $V_s = i_1 R_1 + i_2 R_2 \Rightarrow i_1 = \frac{1}{R_1+R_2} V_s$

Therefore,

$$\boxed{v_o = \left(\frac{R_3}{R_3 + R_4} - \frac{R_1}{R_1 + R_2} \right) V_s} \qquad (\star)$$

Therefore $v_o = 0$ if

$$\frac{R_3}{R_3+R_4} = \frac{R_1}{R_1+R_2}$$

$$R_3(R_1+R_2) = R_1(R_3+R_4)$$

$$\cancel{R_3R_1} + R_3R_2 = \cancel{R_1R_3} + R_1R_4$$

$$R_3R_2 = R_1R_4$$

$$\boxed{\frac{R_3}{R_4} = \frac{R_1}{R_2}} .$$

This is a useful result. When this condition is met, the Wheatstone bridge is said to be balanced.

If one of the resistors — conventionally R_1 — is placed (remotely from the others) in an environment that changes its resistance from its nominal (balanced) value, measuring the output voltage v_o will indicate this change.

Typically one of the following two methods is used.

Null method We can determine the change in resistance of R_1 by "balancing" the bridge: adjusting the resistance of (say) R_4 until the bridge is once again balanced.

Deflection method We can also use the Wheatstone bridge to output a voltage that depends on the change in R_1, δR_1. We can use (*) to determine how v_o depends on δR_1.