# Complex voltages + currents

It is common to represent voltage and current in circuits as complex numbers, especially when sinusoidal signals are present.

Let a voltage $v(t) = v_0 \cos(\omega t + \phi)$ be represented by

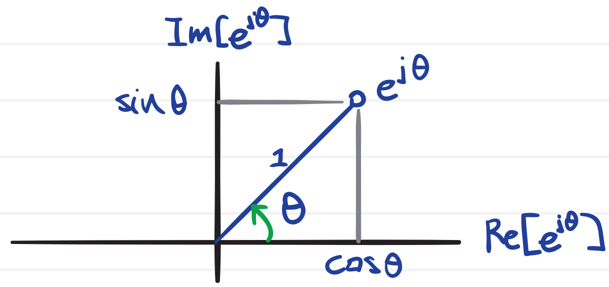
$$\begin{aligned} v(t) &= v_0 \, \mathrm{Re}\left[e^{j(\omega t + \phi)}\right] \\ &= v_0 \, \mathrm{Re}\left[e^{j\phi} e^{j\omega t}\right], \end{aligned}$$

where we have used Euler's formulas

$$\left\{ \begin{aligned} e^{j\theta} &= \cos\theta + j\sin\theta \\ e^{-j\theta} &= \cos\theta - j\sin\theta \\ \cos\theta &= \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) = \mathrm{Re}\left[e^{j\theta}\right] \\ \sin\theta &= \tfrac{1}{2}\left(e^{j\theta} - e^{-j\theta}\right) = \mathrm{Im}\left[e^{j\theta}\right] \end{aligned} \right\} \longleftarrow (*)$$

These formulas describe a vector in the complex plane.

$e^{j\theta} = \cos\theta + j\sin\theta$




We typically use $(*)$ to convert from complex exponential representations to trigonometric ones. We often use shorthand notation with which we "drop" the $\mathrm{Re}[\cdot]$ and $e^{j\omega t}$.

$$\begin{tikzcd} v(t) = v_0 \cos(\omega t + \phi) \arrow[d, "\text{convert}"'] & v'(t) = v_0' \cos(\omega t + \phi') \\ v(t) = v_0 e^{j\phi} \arrow[r, "\text{operate}"'] & v'(t) = v_0' e^{j\phi'} \arrow[u, "\mathrm{Re}[\cdot]"] \end{tikzcd}$$

The same representation is used for alternating current $i(t) = i_0 e^{j\phi}$.
This representation is often called phasor notation. (Sometimes: $a e^{j\phi} = a\angle\phi$.)