Impedance

With complex representations for voltage and current, we can introduce the concept of impedance. Impedance Z is the complex ratio of voltage to current in a circuit element.

$$Z \equiv v/i \quad .$$

The real part is called the resistance and the imaginary part the reactance:

$$\text{impedance} = (\text{resistance}) + j\,(\text{reactance}) \quad .$$

As with complex voltage and current, we can represent the impedance as a phasor.

We can use the impedance to define the generalized Ohm's law:

$$\boxed{v = i\,Z}$$

for each circuit element.

Impedances of circuit elements

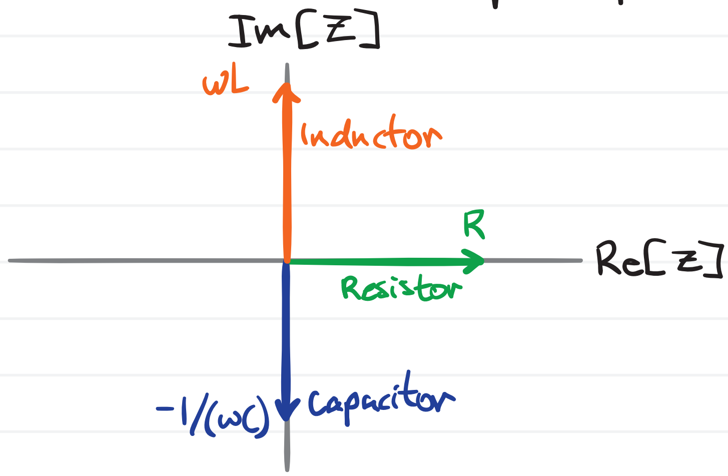
The impedances of the three most common circuit elements are listed below in complex forms. ($\omega \equiv$ angular frequency)

Resistor: $Z = R\cos 0 + j\sin 0 = R = Re^{j0} = R\angle 0$

Capacitor: $Z = \frac{1}{\omega C}\left(\cos\frac{-\pi}{2} + j\sin\frac{-\pi}{2}\right) = 1/(j\omega C) = \frac{1}{\omega C}e^{-j\frac{\pi}{2}} = \frac{1}{\omega C}\angle\frac{-\pi}{2}$

Inductor: $Z = \omega L\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right) = j\omega L = \omega L e^{j\frac{\pi}{2}} = \omega L\angle\frac{\pi}{2}$.

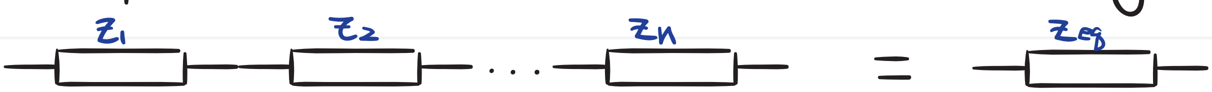
Here is an illustration in the complex plane.



Combining impedances

As with resistors, impedances may be combined to find an effective impedance. We often want to know the effective impedance of a circuit or portion thereof.

Series impedances can be combined in the following manner:



$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_n = \sum_{k=1}^{n} Z_k$$

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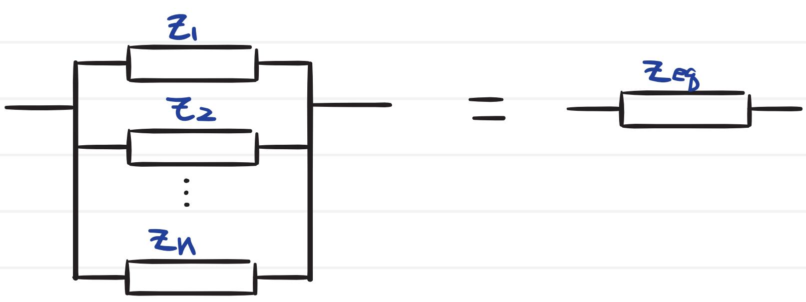
Parallel impedances can be combined in the following manner:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}} = 1 \Big/ \sum_{k=1}^{n} 1/Z_k$$

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