

# Maximum shear stress theory (ductile)

Maximum shear stress theory predicts failure in ductile materials based on the observation that ductile materials tend to yield along slip lines (Lüder lines) that are around  $45^\circ$  from the axis of an axially loaded strip in tension. The maximum shear stress is also nearly  $45^\circ$ . It is, therefore reasonable to assume that failure is caused by shear stress.

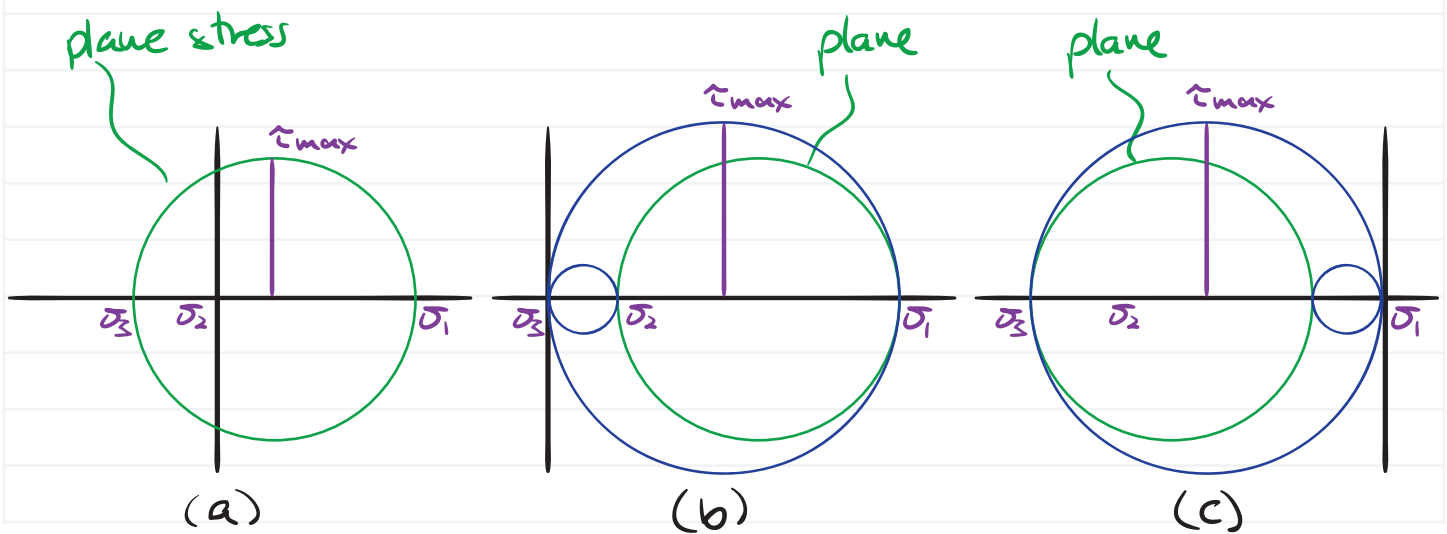
At  $45^\circ$ ,  $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$ . Therefore failure is predicted when

$$\tau_{max} \geq S_y/2 \quad (\text{i.e.}) \quad \sigma_1 - \sigma_3 \geq S_y$$

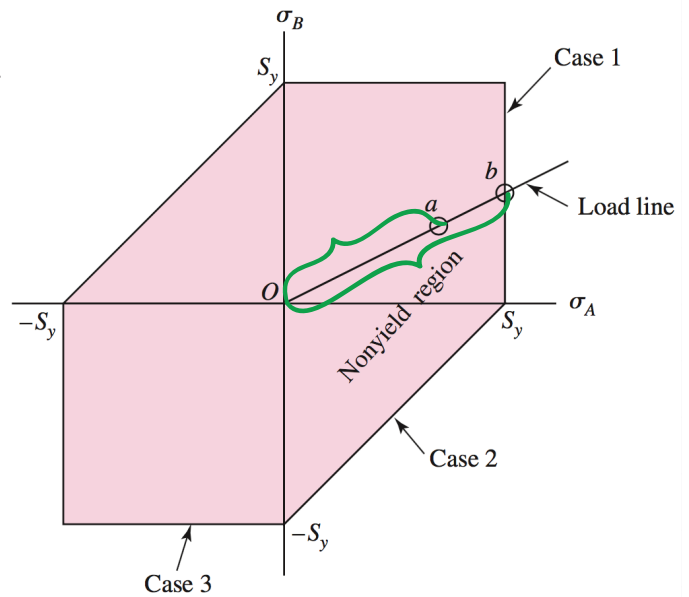
We may incorporate a factor of safety  $n$  as follows: the maximum allowable stress is

$$\tau_{max} = \frac{S_y}{2n} \quad (\text{i.e.}) \quad \sigma_1 - \sigma_3 = S_y/n$$

Recall that we must take care when analyzing plane-stress. The out-of-plane stress is 0, which may be the max or minimum normal stress, in some situations.



For this reason, when we find the normal plane stresses, we should call them  $\sigma_A + \sigma_B$  instead of  $\sigma_1 + \sigma_3$  until we have determined which of the situations (a), (b), or (c), we have.



The nonyield envelope of plane stress is shown at right. Those elements stressed inside the envelope are considered not to fail.

If a state of plane stress  $(\sigma_A, \sigma_B)$  give point a inside the envelope, and the load line is defined to be the line through  $(0,0)$  and  $(\sigma_A, \sigma_B)$ , with its intersection with the edge of the element at point b, then the ratio of lengths

$$n = \frac{Ob}{Oa}$$

is the factor of safety.