Voltage, current, + resistance (adapted from AOE, ch 1)

Two quantities are of special importance in electrical circuits: voltage and current. The relationship between them defines a third important quantity: resistance (or impedance).

We begin by defining each of these.

Voltage

Definition: Voltage is the difference in electrical potential energy of a unit of charge moved
between two locations in an electrical field.
It is typically denoted v, and has units volts with symbol V.

Voltage is always defined by referring to two locations. Sometimes this location is implicitly the ground, which is a reference for a circuit, typically defined to have zero electrical potential energy.

Current

Definition: Current is a flow of charge.
It is typically denoted i, and has units amperes with symbol A.

Some terminology

We generate voltages by doing work on charges with devices like

- batteries (electrochemical work),
- generators (magnetic work), and
- solar cells (photovoltaic conversion).

We get currents by placing voltages across elements, through which current flows.

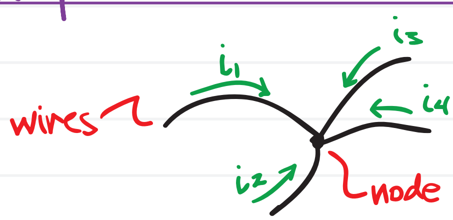
Circuits

Circuits are dynamic electrical systems in which charge accumulates and flows through energy storage elements (e.g. capacitor). Energy storage elements are connected via metalic conductors called wires, which usually (ideally) have the same voltage (relative to, say, ground) everywhere.

Kirchoff's Current Law

Charge is a fundamental physical property that is conserved. This means that, given any point in a circuit that isn't accumulating charge (e.g. wires), the currents flowing in must equal those flowing out.

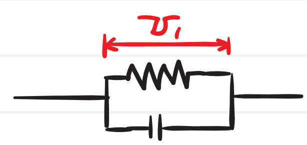
Example



Kirchoff's current law implies that

$$i_1 + i_2 + i_3 + i_4 = 0 \quad \text{or} \quad \sum_{j=1}^{4} i_j = 0 .$$

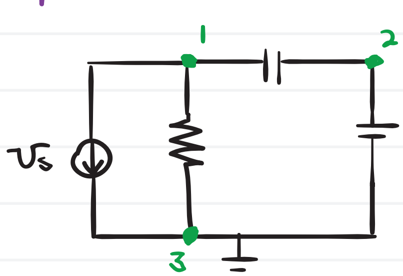
Kirchoff's Voltage Law

Circuit elements connected in parallel, like 

, have the same voltage across them.

Note that this doesn't imply that the current through the elements is the same. However, it does imply that the sum of the voltage drops around any closed loop in a circuit is zero.

Example



Kirchoff's voltage law implies

$$v_{12} + v_{23} + v_{31} = 0 \quad \text{and}$$

$$v_s + v_{31} = 0$$

Power

The instantaneous power being consumed by a circuit element at a given moment is

$$P(t) = v(t)\,i(t).$$

($v(t)$: instantaneous voltage across the element; $i(t)$: instantaneous current through the element)

Power typically goes into:

- heat (usually),
- mechanical work (motors),
- radiated energy (lamps, transmitters), or
- stored energy (batteries, capacitors).

More on terminology

> "[D]on't call current 'amperage'; that's strictly bush-league. The same caution will apply to the term 'ohmage'....
>
> — Horowitz + Hill, The Art of Electronics

Resistors

Roughly: electronics is all about making and using circuit elements with interesting i—v relationships.

Examples:

- resistors (i proportional to v),
- capacitors (i proportional to rate-of-chang of v),
- diodes (i flows in only one direction),
- thermistors (temperature-dependent resistors),
- photoresistors (light-dependent resistors), and
- strain gauges (strain-dependent resistors).

Metal conductors with high conductivity tend to quickly transfer around any concentrations of charge, thus leaving very little potential difference (voltage). We use these as wires.

However some metal conductors with low conductivity have the interesting property that the voltages across them are proportional to the currents through them.

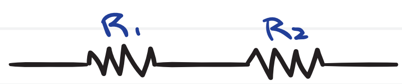
A resistor is made out of these types of conductors (e.g. carbon or thin metal) with a wire coming out of each end. It is characterized by its resistance:

$$R \equiv v/i \ , \leftarrow \text{Ohm's Law}$$

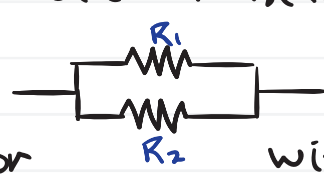
which is often assumed to be constant for certain operating regimes (e.g. temperature) of a given resistor.

R has SI units of Ohms (Ω).

Roughly speaking, resistors are used to convert voltage to current and current to voltage.

Resistors in series 

can be considered to be a single resistor with resistance $R_{eq} = R_1 + R_2$.

Resistors in parallel 

can be considered to be a single resistor with resistance

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2} .$$