Basic probability theory

The sample space Ω of an experiment is the set representing all possible outcomes of the experiment.

If a coin is flipped, the sample space is $\Omega = \{H, T\}$.

If a coin is flipped twice, we can choose .

However, the same experiment can have different sample spaces. For instance for two coin flips, we could choose

We base our choice of Ω on the problem at hand.

An event is a subset of the sample space. That is, an event corresponds to a yes-or-no question about the experiment. For instance, event A (remember: $A \subseteq \Omega$) in the coin flipping experiment (two flips) might be $A = \{HT, TH\}$. A is an event that corresponds to the question, "Is the second flip different than the first?" A is the event for which the answer is "yes."

Algebra of events

Because events are sets, we can perform the usual set operations with them.

Example Consider a toss of a single die. We choose the sample space to be $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let the following define events.

Now we might be interested in combinations of events A and B. Here are some examples:

The event class $\mathcal{F}$ is often defined as the set of all subsets of Ω. (It's actually more complicated, but we'll ignore that.) So $\mathcal{F}$ is the set of all possible events. When referring to an event, we will often state that it is an element of $\mathcal{F}$. (E.g. $A \in \mathcal{F}$.)

We're finally ready to assign probabilities to events. We denote the probability of an event A by $P(A)$ or $Pr(A)$.

We define the probability measure $P: \mathcal{F} \to [0,1]$ (i.e. P is a function from the event class $\mathcal{F}$ to the interval $[0,1]$) to be a function satisfying the following conditions.

The three structures we've defined thus far—Ω (sample space), $\mathcal{F}$ (event class), + P (probability measure)— are called the probability space $(\Omega, \mathcal{F}, P)$

We conclude with the basics by observing four facts that can be proven from the definitions above.

1. $P(\emptyset) = 0$.

3. If $B \subset A$, then $P(B) < P(A)$. In fact, $P(A \setminus B) = P(A) - P(B)$.

4. $P(A_1 \cup A_2 \cup \cdots) \leq P(A_1) + P(A_2) + \cdots$.