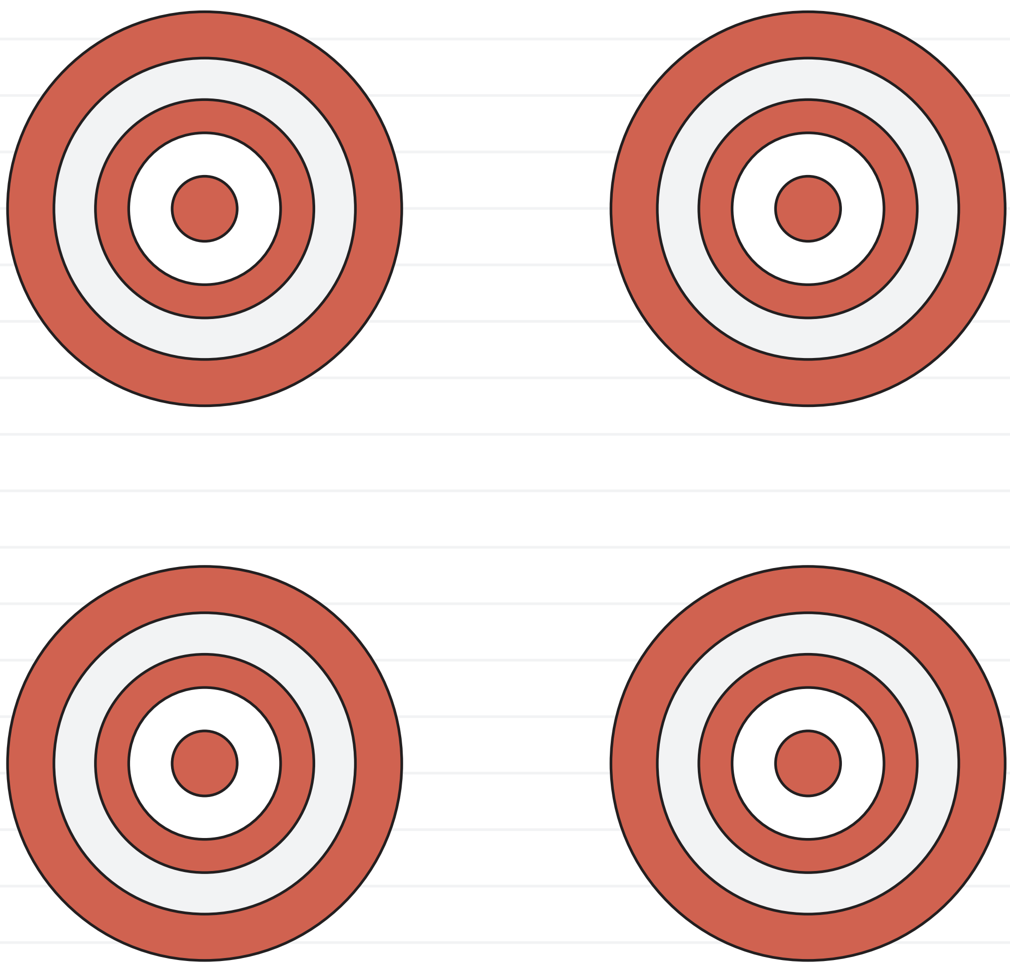
Basic uncertainty analysis

Systematic and random errors

Systematic errors are differences between the measured and "true" value of a measurand. Systematic errors decrease as **accuracy** of a measurement increases. Usually, systematic errors are mitigated by **calibration**.

Random errors characterize the scatter of repeated measurements of a measurand. These errors decrease as **precision** of a measurement increases. Usually, random errors are mitigated by larger samples (**repetition**).



Quantifying systematic and random errors

Let a measurand be represented by r.v. X.
The random uncertainty (precision limit) of the measured value x is

where S_{P_x} is the sample std of the random error and $t_{\nu_{P_x},C}$ is the student's t-variable with $\nu_{P_x} = N-1$, where N is the sample size and C is the confidence level.

Rearranging (*), $S_{B_x} = B_x / t_{\nu_{B_x},C}$. So we can compute the std of the systematic error for a given confidence. For instance, $S_{B_x} \cong B_x/2$ with 90% confidence.

The degrees of freedom $\nu_{B_x} \cong \frac{1}{2}\left(\frac{\Delta B_x}{B_x}\right)^{-2} = \frac{1}{2}\left(\frac{\Delta S_{B_x}}{S_{B_x}}\right)^{-2}$, where ΔB_x is the reliability of the measurement. Instruments often have systematic error B_x specified by the manufacturer. ΔB_x is the interval over which we trust B_x.

Example A manufacturer of multimeters reports an accuracy of 1 mV. We aren't particularly confident in this, and so we make measurements and find that the reliability of this value is 0.5 mV. What is the relative systematic uncertainty of the 1 mV accuracy?