Central Moments

Once we have determined the PDF/PMF for a r.v., a number of useful parameters of the r.v. can be determined. These are computed by taking central moments of the PDF/PMF.

Central moments quantify parameters of the r.v. relative to its expected value ("center").

Before we define these moments, we give a familiar name to the expectation.

Definition The **mean** of a random variable X is defined as

Now we can define central moments.

Definition The n^{th} central moment of r.v. X (with PDF $f(x)$) is defined as

$$\langle (X - m_X)^n \rangle = \int_{-\infty}^{\infty} (X - m_X)^n f(x) dx .$$

For discrete r.v. K (with PMF $f[k]$),

Class Exercise Prove that the first central moment is 0.

Higher central moments: variance, skewness, and kurtosis

The variance is the second central moment:

The variance is a measure of the width or spread of the PD(M)F. We usually compute the variance with the formula

$$\mathrm{Var}[X] = \langle X^2 \rangle - m_X^2 .$$

Other properties of the variance include the following.

$$\mathrm{Var}[c] = 0 \qquad \text{(c a constant)}$$
$$\mathrm{Var}[X + c] = \mathrm{Var}[X]$$
$$\mathrm{Var}[cX] = c^2 \mathrm{Var}[X] .$$

The standard deviation is defined as

It is often used for error bars on plots.

The skewness is based on the third central moment:

Skewness is a measure of assymetry of the PDF of X. For a symmetric PDF, $\alpha_3 = 0$.

The kurtosis is based on the fourth central moment:

$$\alpha_4 = \frac{\langle (X - m_X)^4 \rangle}{\sigma^4} - 3 .$$