Dynamic response of measurement systems: an introduction

Measurement systems are dynamic systems. By design they respond to their environment — ideally, the measurand. In order to properly understand how these measurement systems can be used, we must understand dynamic system response. This is true of sensors, transducers, and signal conditioners.

For single-input, single-output (SISO) linear systems, we can write the following ODE to describe its dynamics.

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where $y(t)$ is the output, $f(t)$ is the forcing function, and $a_i \in \mathbb{R}$ are coefficients that depend on the system parameters.

This ODE can be solved using various methods, and the solution is guaranteed to exist and be unique by the existence and uniqueness theorem.

We have already considered a number of systems with dynamical equations of this form. Many measurement systems can be modeled as zeroth-, first-, or second-order.

Because this is not a course on system dynamics, we will proceed by example, in an attempt to call to mind along the way methods learned previously.

First order response to sinusoidal forcing

Let's consider a first-order system with a sinusoidal forcing function. This sort of system is described by the ODE $\tau \dot{y} + y = A \sin \omega t$. It could be a capacitive sensor or a low- or high-pass filter, for example. Let's consider the time-response of such a system.

<u>Solving the ODE</u> The general solution is $y(t) = y_h(t) + y_p(t)$, where $y_h(t)$ is the homogeneous solution and $y_p(t)$ is the particular solution. The general solution is a family of all possible solutions. Given a set of initial conditions, the specific solution $y_s(t)$ can be selected from the family of solutions. This is true of all orders of systems and inputs, and we use it here in the first order case.

1. Homogeneous solution:

2. Particular solution:

3. General solution:

4. Specific solution: apply the initial condition $y(0) = y_0$:

c. Putting it together:

with

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<u>Further analysis</u> The term $\left(y_0 + \frac{\tau\omega A}{\sqrt{1+\tau^2\omega^2}}\right)e^{-t/\tau}$ decays with time, and so we call it the transient response. The other term doesn't decay, so we call it the steady-state response.

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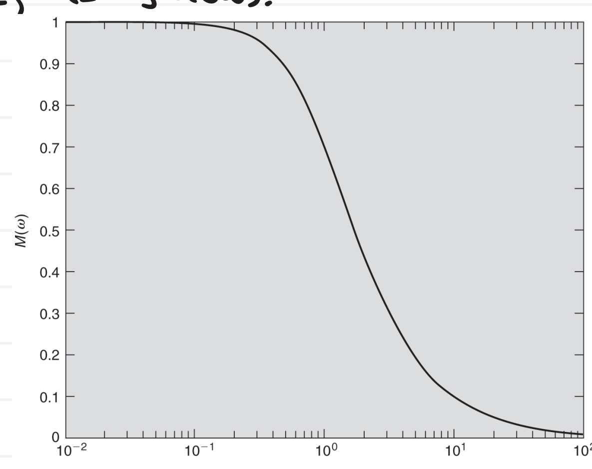
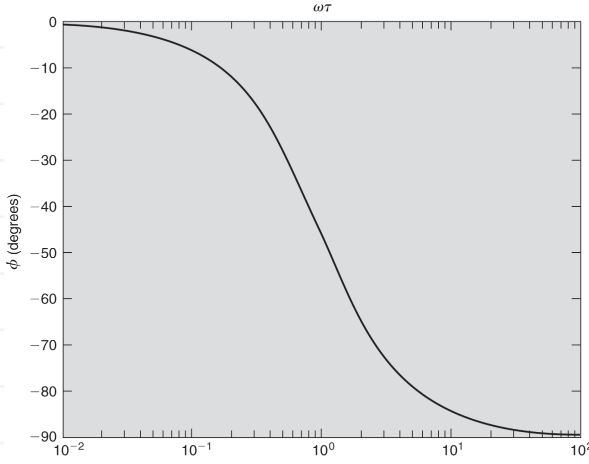
The term $\tau\omega$ is dimensionless, so we often use it when we plot the magnitude ratio and phase, as follows.





Below, we plot an example of a response like we've been analyzing.

β is the time lag:

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