Expectation

A random variable is a function that maps from the sample space to the real numbers, $X: \Omega \rightarrow \mathbb{R}$. The variables that probability mass functions (PMF) and probability density functions (PDF) accept as arguments are random variables.

Random variables can have an expected value (or expection). This can be thought of as the "average value" of the random variable.

The expected value of r.v. X is denoted $\langle X \rangle$ or $E[X]$.

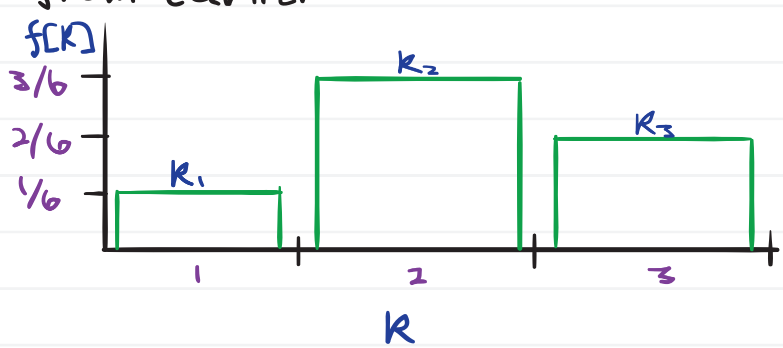
Expectation of discrete random variables

Definition: Let K be a discrete random variable with PMF $f[k]$. We define the expectation of K to be

$$\langle K \rangle \equiv \sum_{\forall k} k f[k] .$$

Here's an example.

Example Recall the PMF from earlier:



What is the expected value of K?

$\langle K \rangle = \Sigma\, k_i f[k_i]$

Expectation of continuous random variables

Definition Let X be a continuous r.v. with PDF $f(x)$. We define the expectation of X to be

$$\langle X \rangle = \int_{-\infty}^{\infty} x f(x)\, dx .$$

Example Given a continuous r.v. X with a Gaussian PDF $f(x)$, what is the expected value of X?

Recall that $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-(x-\mu)^2/(2\sigma^2)\right)$. Then

Note that in general, integrating over the Gaussian PDF is difficult, and the procedure in Section 12.3 of Dunn is typically used (or a mathematical software tool like Matlab). Read it!

Properties of the expectation operator

The expectation operator is linear. In other words, it obeys

$$\langle aY + bZ \rangle = a\langle Y \rangle + b\langle Z \rangle .$$

Here are a few others for r.v. X and real number c.