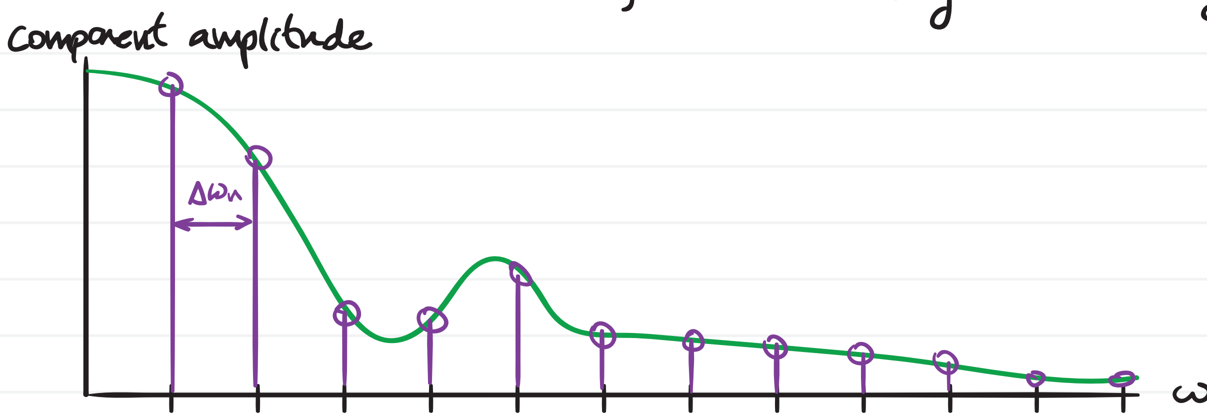
Fourier transform

Aperiodic signals do not have a Fourier series representation, but they do have a Fourier transform representation. The Fourier transform is derived from considering an infinite number of harmonics, which is equivalent to taking the limit as the period $T \to \infty$.

Here are the fundamental equations for the trigonometric form of the Fourier transform representation of a function $y(t)$.

We can visualize the differences and similarities of the Fourier series and Fourier transforms in the gfollowin way.



The Fourier transform may need all frequencies, not just integer multiples of the fundamental ω_1.

Complex Fourier transform

It is more common to represent the Fourier transform in complex form. The complex Fourier coefficient is

This gives rise to the Fourier transform pair

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The Fourier transform is more useful than the Fourier series. It can be used to characterize the response of systems and solve differential equations. Perhaps most useful is the idea that each time-domain signal $y(t)$ has an equivalent frequency-domain representation $Y(\omega)$.

Properties of the Fourier transform

See the resource:

http://homepages.stmartin.edu/fac_staff/RPicone/courses/me315/resources/FourierTransformTable.pdf

This table summarizes a number of important Fourier transforms and properties

Example What is the Fourier transform of

$$f(t) = \begin{cases} K e^{-at} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}.$$

Use both the definition and the table.

Solution From the definition,

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

From the table,