Notes on Generalized Impedances by J. L. Garbini

Generalized impedances are an extension of the concept of electrical impedances to systems of other domains. The table below lists the corresponding driving-point impedance definitions for five different energy modalities.

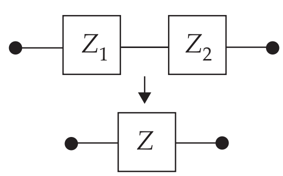
		Mechanical Translational	**Mechanical Rotational**	**Electrical**	**Fluid**	**Thermal**
Across Variable		v, velocity	ω, angular velocity	v, voltage	p, pressure	T, temperature
Through Variable		f, force	T, torque	i, current	q, volumetric flow	q, heat flow rate
Impedance $Z(s)$ **Admittance** $Y(s) = \frac{1}{Z(s)}$		$Z(s) = \frac{V(s)}{F(s)}$	$Z(s) = \frac{\Omega(s)}{T(s)}$	$Z(s) = \frac{V(s)}{I(s)}$	$Z(s) = \frac{P(s)}{Q(s)}$	$Z(s) = \frac{T(s)}{Q(s)}$
Impedance Z(s)	**A-Type**	mass, M: $\frac{1}{Ms}$	inertia, J: $\frac{1}{Js}$	capacitor, C $\frac{1}{Cs}$	fluid capacitor, C $\frac{1}{Cs}$	thermal capacitor, C $\frac{1}{Cs}$
	D-Type	damper, B $\frac{1}{B}$	r. damper, B $\frac{1}{B}$	resistor, R R	fluid resistor, R R	thermal resistor, R R
	T-Type	spring, K $\frac{s}{K}$	r. spring, K_r $\frac{s}{K_r}$	inductor, L Ls	fluid inductor, L Ls	—

Series and parallel combinations of impedances and admittances can be combined. In the following V and F represent the across and through variables respectively of any physical domain.

Series Combination

Elements sharing a common *through* variable are in *series*.

The *impedance* of elements connected in *series* is the sum of the individual *impedances*.

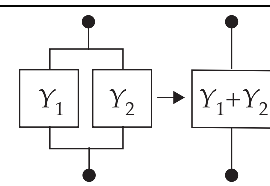


$$Z_1 = Z_1 + Z_2$$

Parallel Combination

Elements sharing a common *across* variable are in *parallel*.

The *admittance* of elements connected in *parallel* is the sum of the individual *admittances*.



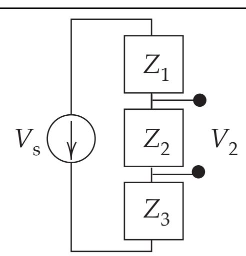
$$Y = Y_1 + Y_2$$

$$Z = \frac{1}{Y} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Simple transfer functions can be determined from impedance/admittance properties.

Across Variable Divider

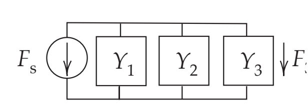
The complex amplitude of the *across variable* across a set of elements in *series* is divided among the elements in proportion their *impedances*.



$$T(s) = \frac{V_2(s)}{V_s(s)} = \frac{Z_2}{Z_1 + Z_2 + Z_3}$$

Through Variable Divider

The complex amplitude of the *through variable* through a set of elements in *parallel* is divided among the elements in proportion their *admittances*.



$$T(s) = \frac{F_3(s)}{F_s(s)} = \frac{Y_3}{Y_1 + Y_2 + Y_3}$$

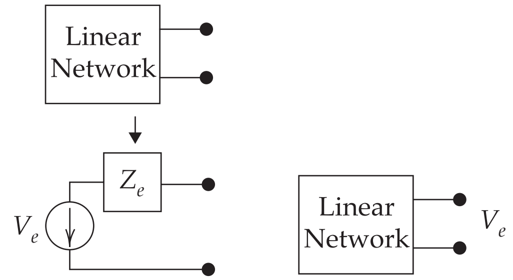
Thevenin and Norton equivalent networks are useful deriving transfer functions and in modeling systems that have a defined load impedance.

Thevenin's Theorem

A linear two-terminal network is equivalent to an across variable source V_e in *series* with an equivalent impedance Z_e, where

Z_e = the impedance of the network with all sources set equal to zero, and

V_e = an *across variable source* equal to the across variable that would appear across the *open* circuit terminals of the network.

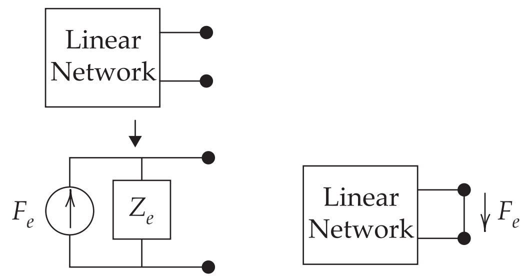


Norton's Theorem

A linear two-terminal network is equivalent to a through variable source F_e in *parallel* with an equivalent impedance Z_e, where

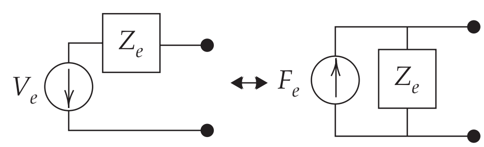
Z_e = the impedance of the network with all sources set equal to zero, and

F_e = a *through variable source* equal to the through variable that would flow through the *short* circuited terminals of the network.



Source Transformations

Since any linear two-terminal networks can be represented by either a Thevenin equivalent or a Norton equivalent, the two representations must be equivalent to each other.



$$F_e = \frac{V_e}{Z_e}$$

Note: In both Thevenin and Norton networks the impedance Z_e is determined by finding the impedance at the terminals with all of the sources set equal to zero. The sources are set to zero, as follows:

- **Across Variable Source**: *Replace* the across variable source with a short circuit. That is, the nodes to which the source is connected are joined together.
- **Through Variable Source**: *Remove* the through variable source from the network.

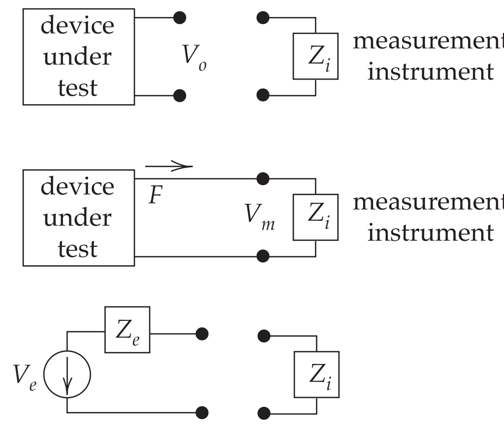
Measurement Loading

Across Variable Measurements

Suppose that we wish to measure an across variable at the output of a "device under test" with a "measurement instrument." The measurement instrument is attached across the terminals of interest. Of course we desired that the measured variable be undisturbed by the connection of the instrument. That is, we want V_m to be as nearly equal to V_o as possible. We say that the measurement instrument should not "load" the device under test.

The *output impedance* of the device under test is the equivalent impedance defined by its Thevenin model $Z_o = Z_e$ for the unloaded output (disconnected) terminals.

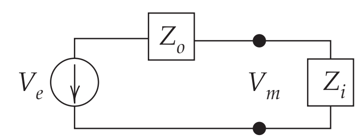
Similarly, the *input impedance* Z_i of the measurement instrument is the Thevenin equivalent impedance defined for its input terminals.



Connecting the Thevenin model for the device under test to the input impedance of the measurement instrument we have the network at the right.

The Thevenin equivalent across variable source is by definition equal to V_o, the value that we wish to measure. Applying the across variable divider rule: $\frac{V_m(s)}{V_o(s)} = \frac{1}{1+Z_o/Z_i}$.

Since we desire that the ratio approach unity, the input impedance of the measurement instrument must be large in comparison with the output impedance of the device under test: $Z_i >> Z_o$

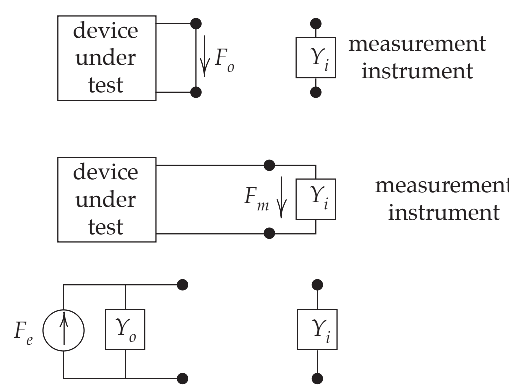


Through Variable Measurements

Alternately, suppose that we wish to measure a through variable in a device under test with a measurement instrument. In this case, the variable of interest flows through the measurement instrument. We desired that the measured variable be undisturbed by the connection of the instrument. That is, we want F_m to be as nearly equal to F_o as possible.

The *output admittance* of the device under test is the equivalent admittance defined by its Norton's model $Y_o = 1/Z_e$ for the unloaded (disconnected) output terminals.

Similarly, the *input admittance* Y_i of the measurement instrument is the Norton equivalent admittance defined for its input terminals.



Connecting the Norton model for the device under test to the input admittance of the measurement instrument we have the network at the right.

The Norton equivalent through variable source is by definition equal to F_o, the value that we wish to measure. Applying the through variable divider rule: $\frac{F_m(s)}{F_o(s)} = \frac{1}{1+Y_o/Y_i}$.

Since we desire that the ratio approach unity, the input admittance of the measurement instrument must be large in comparison with the output admittance of the device under test: $Y_i >> Y_o$