Independence

Two events A and B are independent iff

Independence of two events is determined by the experimenter when the experimenter has no reason to believe the events are dependent.

Example Consider a single, fair die rolled two consecutive times. What is the probability that both rolls are "6"?

Conditional probability

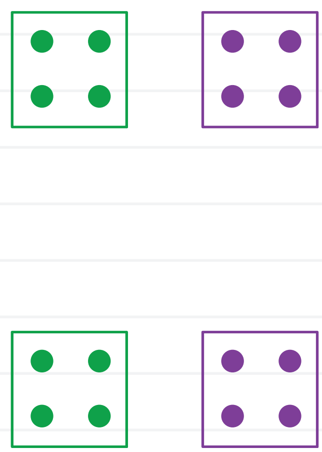
If events A and B are somehow dependent, we need a way to compute the probability of B occurring given that A has already occurred. This is called the conditional probability of B given A, and is denoted $P(B|A)$. It is defined as

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad \text{(assuming } P(A)>0\text{)}.$$

Example Consider two unbiased dice rolled once. Let events $A = \{\text{sum of faces} = 8\}$ and $B = \{\text{faces are equal}\}$. What is the probability that the faces are equal given that their sum is 8?

When we computed $P(A)$, we were concerned with which die each number came from. These types of problems are called those for which "order matters." We counted the outcomes $5+3$ and $3+5$ as distinct, for instance. However, we didn't count $4+4$ twice. Why not?

There is no distinction between $4+4$ and $4+4$. Here's an illustration:



Bayes' Theorem (Rule)

Given two mutually exclusive and exhaustive events A and B, Bayes' Theorem states that

Sometimes this is written:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

$$= \frac{1}{1 + \frac{P(B|A')}{P(B|A)} \cdot \frac{P(A')}{P(A)}}.$$

This is a useful theorem for determining a test's effectiveness.

If a test is performed to determine whether an event has occurred, we might ask questions like "If the test indicates that the event has occurred, what is the probability it actually has occurred? Bayes' rule can help compute an answer.

Four types of outcomes can occur:

1. True positive: test indicates occurrence and actual occurrence.
2. False positive: test indicates occurrence but no actual occurrence.
3. False negative: test indicates no occurrence but actual occurrence.
4. True negative: test indicates no occurrence and no actual occurrence.

	True	False
Positive		
Negative		

↑ As measurement engineers, we want to live in the first column as much as possible. Good tests yield 1 and 4 most of the time. Poor tests yield 2 and 3 too often.

Example Suppose $x \in [0,1]$ of springs manufactured at a given plant are defective. Suppose you need to design a test that has probability of $y \in [0,1]$ that a part indicated defective is actually so. How accurate must your test be? (i.e. what probability should your test have of indicating an actually defective spring?)

Define the events $A = \{\text{spring defective}\}$ and $\hat{A} = \{\text{spring indicated defective}\}$. Then $P(A) = x$ and the spec requires that $P(A|\hat{A}) = y$. We need to compute $z \equiv P(\hat{A}|A)$. Because A and $\hat{A}$ are mutually exclusive and exhaustive, $P(A') = 1 - P(A) = 1 - x$ and $P(\hat{A}|A') = 1 - P(\hat{A}|A) = 1 - z$.

Now we can apply Bayes' Theorem

$$P(A|\tilde{A}) = \frac{P(\tilde{A}|A)\,P(A)}{P(\tilde{A}|A)\,P(A) + P(\tilde{A}|A')\,P(A')}.$$

Solving for z,

Finally, we can graph the results.