3.

20% overshoot => $\zeta = 0.456$ => $\phi_M = 48.15°$.

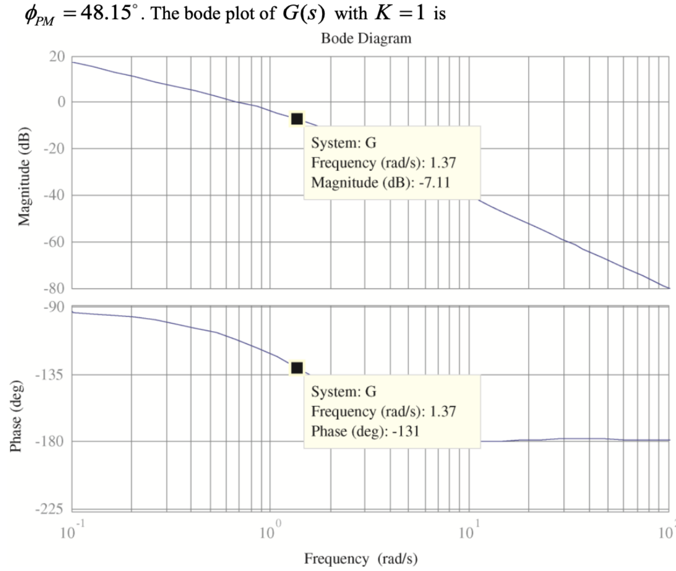
a. Looking at the phase diagram, where $\phi_M = 48.15°$ (i.e. $\phi = -131.85°$), the phase margin frequency = 4.11 rad/s. At this frequency, the magnitude curve is -55.2 dB. Thus the magnitude curve has to be raised by 55.2 dB (K = 575).

b. Looking at the phase diagram, where $\phi_M = 48.15°$ (i.e. $\phi = -131.85°$), the phase margin frequency = 7.14 rad/s. At this frequency, the magnitude curve is – 65.6 dB. Thus the magnitude curve has to be raised by 65.6 dB (K = 1905).

c. Looking at the phase diagram, where $\phi_M = 48.15°$ (i.e. $\phi = -131.85°$), the phase margin frequency = 8.2 rad/s. At this frequency, the magnitude curve is – 67.3 dB. Thus the magnitude curve has to be raised by 67.3 dB (K = 2317).

4.

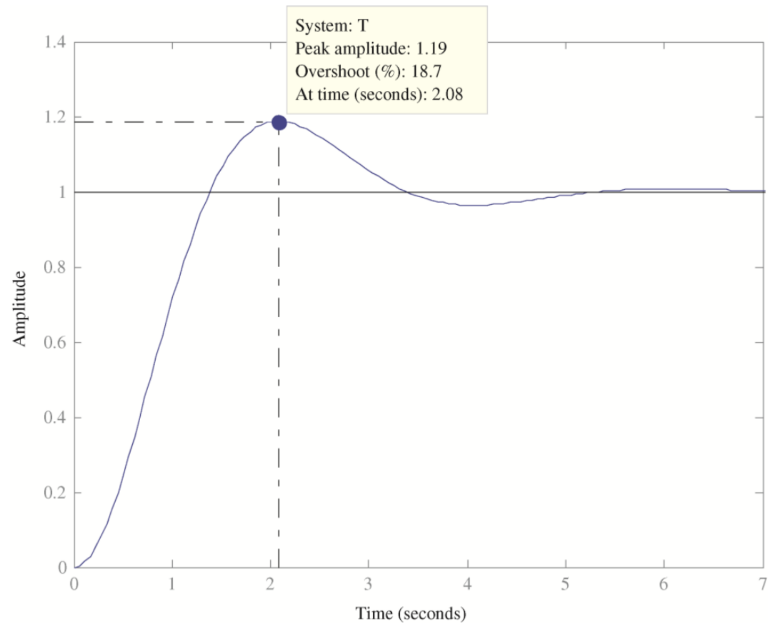
a. A 20% overshoot corresponds to a damping factor of $\zeta = 0.456$ and from equation 10.73 to a $\phi_{PM} = 48.15^\circ$. The bode plot of $G(s)$ with $K = 1$ is



We search on the Bode diagram for the frequency at which the phase is $-180^\circ + 48.15^\circ = -131.85^\circ$. This occurs at a frequency of 1.37 rad/sec. At this frequency $|G(j1.37)| = -7.11$ dB. Thus the magnitude characteristic must be raised by this quantity; $K = 10^{\frac{7.11}{20}} = 2.267$.

b.

```
>> s=tf('s');
>> G=2.267*(s+10)*(s+15)/s/(s+2)/(s+5)/(s+20);
>> T=feedback(G,1);
>> step(T)
```



14.

The following MATLAB M-file was written to assist in solving this problem:

```
'G(s)'                                % Display label.
G=zpk(K*Gp)                      % Define G(s), put K into it,
                                 % convert
                                 % to factored form, and display.
Pm=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
                                 % Calculate phase margin.
Tsd= input('Type Desired Settling Time, Tsd=(Ts/2) = ');
                                 % Input Desired Ts.
wn=4/(z*Tsd);                    % Calculate required natural
                                 % frequency.
wBW=wn*sqrt((1-2*z^2)+sqrt(4*z^4-4*z^2+2));
                                 % Determine required bandwidth.
w=0.01:0.5:1000;                 % Set range of frequency from 0.01
                                 % to 1000 in steps of 0.5.
bode(G)                          % Display the Bode plots.
pause
[M,P]=bode(G,w);                 % Get Bode data.
[Gm,Pm,Wcg,Wcp]=margin(G);       % Find current phase margin.
Pmreq=atan(2*z/(sqrt(-2*z^2+sqrt(1+4*z^4))))*(180/pi);
                                 % Calculate required phase margin.
Pmreqc=Pmreq+10;                 % Add a correction factor of 10
                                 % degrees.
Pc=Pmreqc-Pm;                    % Calculate phase contribution
                                 % required from lead compensator.
% Design lead compensator
beta=(1-sin(Pc*pi/180))/(1+sin(Pc*pi/180));
                                 % Find compensator beta.
magpc=1/sqrt(beta);              % Find compensator peak
                                 % magnitude.
```

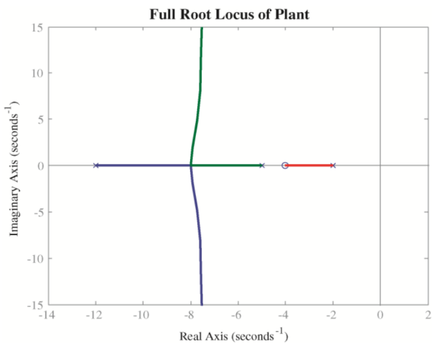
```
for k=1:1:length(M);              % Find frequency at which
                                  % uncompensated system has a
                                  % magnitude of 1/magpc.
                                  % This frequency will be the new
                                  % phase margin frequency.
if M(k)-(1/magpc)<=0;             % Look for peak magnitude.
wmax=w(k);                        % This is the frequency at the
                                  % peak magnitude.
break                             % Stop the loop.
end                               % End if.
end                               % End for.
% Calculate lead compensator zero, pole, and gain.
zc=wmax*sqrt(beta);               % Calculate the lead compensator's
                                  % low break frequency.
pc=zc/beta;                       % Calculate the lead compensator's
                                  % high break frequency.
Kc=1/beta;                        % Calculate the lead compensator's
                                  % gain.
'Gc(s)'                           % Display label.
Gc=tf(Kc*[1 zc],[1 pc]);          % Create Gc(s).
Gc=zpk(Gc)                        % Convert Gc(s) to factored form
                                  % and display.
'Ge(s)=G(s)Gc(s)'                 % Display label.
Ge=G*Gc                           % Form Ge(s)=Gc(s)G(s).
Ge=minreal(Ge);                   % Cancel common factors.
Kp=dcgain(Ge)                     % Calculate Kp.
T=feedback(Ge,1);                 % Find T(s).
step(T)                           % Generate closed-loop, lead-
                                  % compensated step response.
title('Lead-Compensated Step Response')
                                  % Add title to lead-compensated
                                  % step response.
pause
```

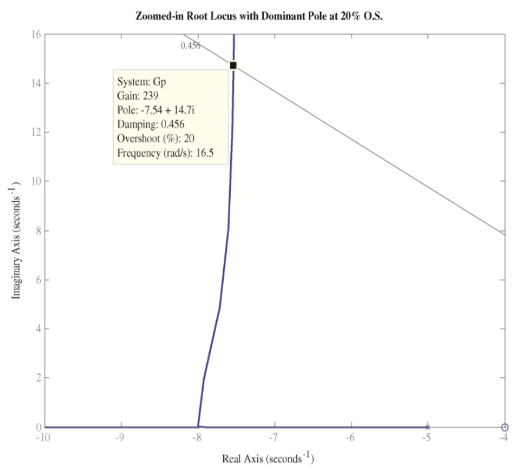
Uncompensated System: The full root locus for the uncompensated system, $G_p(s)$, with the gain set to unity, is shown below. Searching the zoomed-in locus (shown below the full locus) along the ζ = 0.456 line (20% O.S.), find the dominant pole Q = –7.54 + j 14.7 with a gain of 239.

Hence:

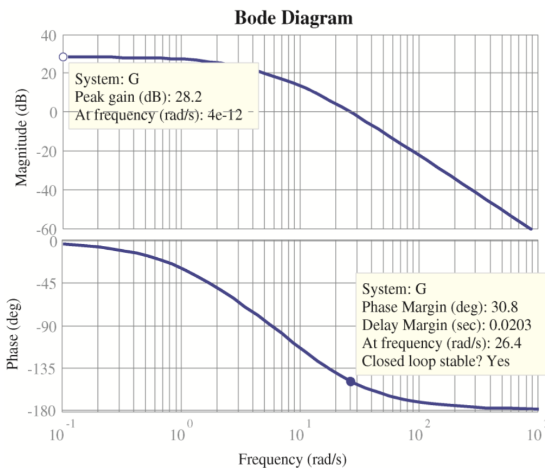
a. $T_s = 4 / \zeta\,\omega_n = 4 / (0.456 \times 16.5) = 0.532$ sec;

b. $Kp = \dfrac{239 \times 4}{2 \times 5 \times 12} = 7.967$.





c. The Bode plot for the uncompensated system is shown below. From that plot we see that the phase margin and the phase-margin frequency are 30.8 degrees and 26.4 rad/sec, respectively.



d. As could be seen from the above M-file, frequency response techniques were used to design a compensator that would yield a threefold improvement in K_P and a twofold reduction in settling time while keeping the overshoot at 20%. That lead compensator was found to have a transfer function:

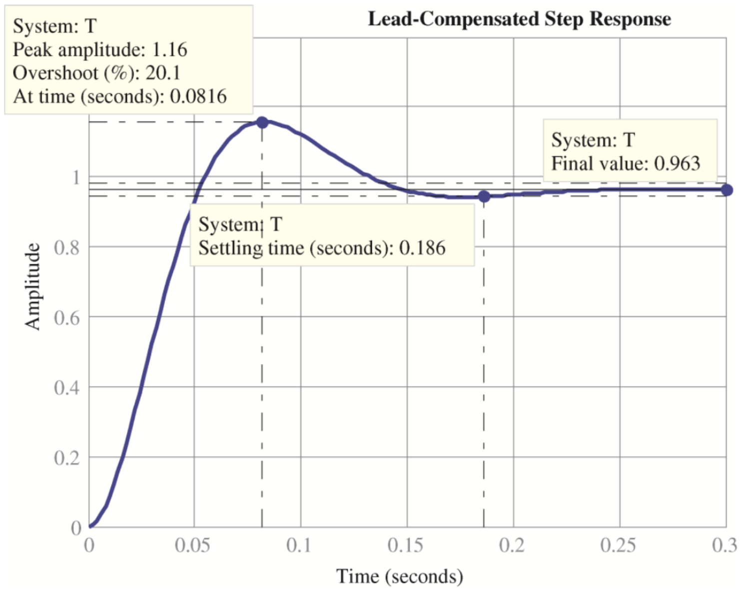
$$G_C(s) = \frac{2.6961(s+21.32)}{(s+57.49)}$$

Compensated system:

Thus, the compensated forward path was found to be:

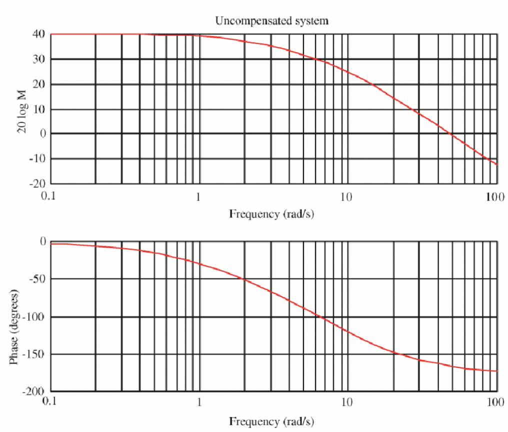
$$G_e(s) = G(s)\,G_C(s) = \frac{2090.1(s+4)(s+21.32)}{(s+2)(s+5)(s+12)(s+57.49)}$$

The step response of the lead-compensated system, shown below, indicates that all requirements are met: We obtained more than threefold improvement in the error constant (K_P was increased more than 3.2 times to 25.84) and the settling time was reduced to 0.186 seconds (almost 2.9 fold reduction) while the overshoot was kept almost at 20%.

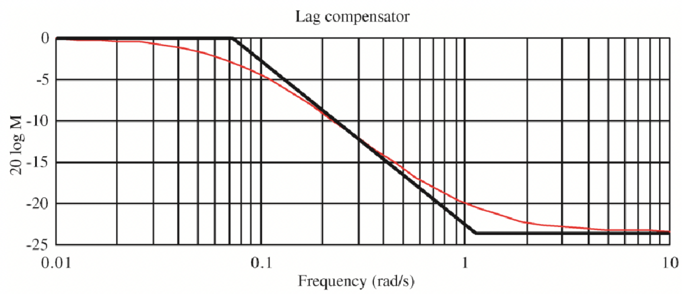


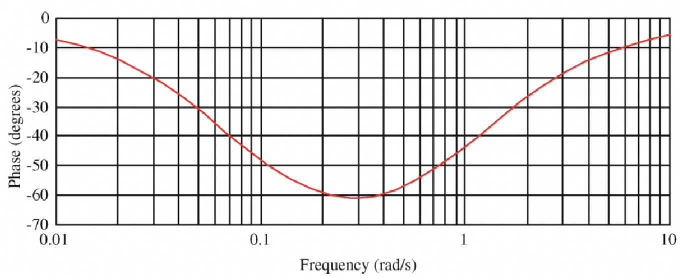
8.

For $K_p = 100 = \dfrac{K(4)}{(2)(6)(8)}$, K = 2400. Plotting the Bode plot for this gain,



We will design the system for a phase margin 10^0 larger than the specification. Thus ϕ_m = 55^0. The phase margin frequency is where the phase angle is $-180^0 + 55^0 = -125^0$. From the Bode plot this frequency is $\omega_{\phi_m} = 11$ rad/s. At this frequency the magnitude is 23.37 dB. Start the magnitude of the lag compensator at –23.37 dB and draw it to 1 decade below $\omega_{\phi_m} = 11$, or 1.1 rad/s. Then begin a +20 dB/dec climb until 0 dB is reached. Read the break frequencies as 0.0746 rad/s and 1.1 rad/s from the Bode plot as shown below.





Ensuring unity dc gain, the transfer function of the lag is $G_{lag}(s) = 0.06782\dfrac{(s+1.1)}{(s+0.0746)}$. The compensated forward-path transfer function is thus the product of the plant and the compensator, or

$$G_e(s) = \frac{162.8(s+4)(s+1.1)}{(s+2)(s+6)(s+8)(s+0.0746)}$$