05

General and specific solutions

We posited in Lecture 02 that the *general solution* to the ODE Equation 01.1 is

general solution

$$y_g(t) = y_h(t) + y_p(t). \tag{05.1}$$

We have not and will not prove this, but simply propose it to be the case. Working through a proof of this from, for instance, your differential equations textbook is of some value.

So, we already have y_h and y_p, so finding y_g is trivial. What type of object is y_g? The particular solution contributes only determined coefficients, but the homogeneous solution contributes n "unknown" constants C_i. This means y_g inherits those constants and therefore is a *family* of solutions.

This leads us to our final step: applying the initial conditions to find the specific constants C_i and thereby our *specific solution* y.

specific solution

"Applying" the initial conditions is simply to subject y_g to each of them. For instance, if we have two initial conditions, such as

we construct two algebraic equations

which is a system of algebraic equations from which the two unknown constants C_1 and C_2 (from the homogeneous solution) can be solved.

Example 05.00-1 A general and a specific solution

Find the general solution for the equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t),$$

with

$$f(t) = a\cos(\omega t),$$

where $a \in \mathbb{R}$ and $\omega = 5$ rad/s. Apply the following initial conditions to obtain a specific solution:

$$y(0) = 3 \quad \text{and} \left.\frac{dy}{dt}\right|_{t=0} = 0.$$

05.01 Exercises

See Appendix A for answers to the following exercises.

In all the following exercises, find the specific solution y for Equation 01.1 with the order n, coefficients a_i, forcing function f, and initial conditions given. Note that the homogeneous and particular solutions from Lecture 04 apply to these problems, so they need not be re-derived.

1. $n = 2$, $a_1 = -1$, $a_0 = -2$, $f(t) = 3$, $y(0) = 2$, $dy/dt|_{t=0} = 0$
2. $n = 2$, $a_1 = 6$, $a_0 = 9$, $f(t) = 5e^{-3t}$, $y(0) = 0$, $dy/dt|_{t=0} = 0$
3. $n = 1$, $a_0 = 2$, $f(t) = 2\cos(3t)$, $y(0) = 4$
4. $n = 3$, $a_2 = 5$, $a_1 = 16$, $a_0 = 80$, $f(t) = t + 2$, $y(0) = 0$, $dy/dt|_{t=0} = 1$, $d^2y/dt^2|_{t=0} = 0$