

## 01.2 intro.sdet State-determined systems

**1** A **system** is defined to be a collection of objects and their relations contained within a **boundary**. The collection of those objects that are external to the system and yet interact with it is called the **environment**. **System variables** are variables that represent the behavior of the system, both those that are internal to the system and those that are external—that is, with the system’s environment.

**2** There are three important classes of system variable, all typically expressed as vector-valued functions of time  $t$ , conventionally all expressed as column-vectors (and called “vectors” even though they’re vector-valued functions ...because nothing makes sense and we’re all going to die). Consider [Figure sdet.1](#) for the following definitions. **Input variables** are system variables that do not depend on the internal dynamics of the system; for a system with  $r$  input variables, the “**input vector**” is a vector-valued function  $\mathbf{u} : \mathbb{R} \rightarrow \mathbb{R}^r$ . The environment prescribes inputs, making them *independent variables*. Conversely, **output variables** are system variables of interest to the designer; for a system with  $m$  output variables, the “**output vector**” is a vector-valued function  $\mathbf{y} : \mathbb{R} \rightarrow \mathbb{R}^m$ . Outputs may or may not directly interact with the environment. Finally, a minimal set of variables that define the internal state of the system are defined as the **state variables**; for a system with  $n$  state variables, the “**state vector**” is a vector-valued function  $\mathbf{x} : \mathbb{R} \rightarrow \mathbb{R}^n$ .

**3** We consider a very common class of system: those that are **state-determined**, which are those for which (Rowell and Wormley, 1997)

1. a mathematical description,
2. the state at time  $t_0$ , called the **initial condition**  $\mathbf{x}(t)|_{t=t_0}$ , and
3. the input  $\mathbf{u}$  for all time  $t \geq t_0$

are necessary and sufficient conditions to determine  $\mathbf{x}(t)$  (and therefore  $\mathbf{y}(t)$ ) for all  $t \geq t_0$ .

4 The “mathematical description” of the system requires a set of primitive elements be assigned to represent its internal and external interactions. The equations derive from two key types of mathematical relationships:

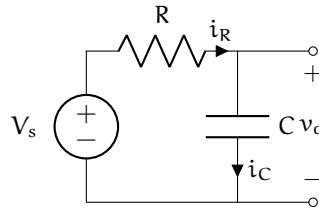
1. the input-output behavior of each primitive element and
2. the topology of interconnections among elements.

The former generate **elemental equations** and the latter, **continuity** or **compatibility equations**.

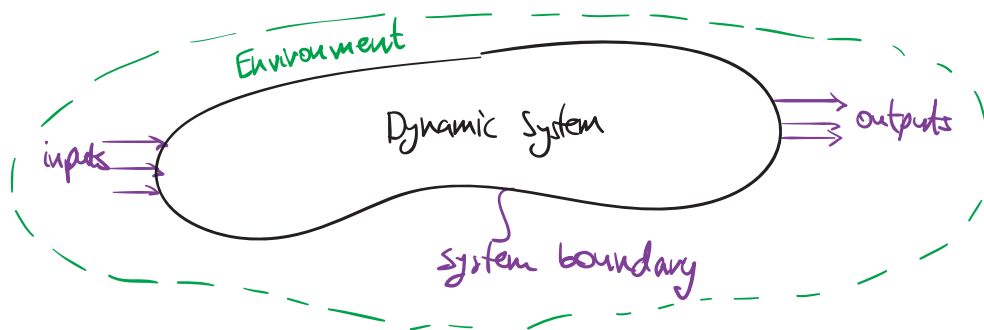
### Example 01.2 intro.sdet-1

In the RC circuit shown, let  $V_s$  be a source and  $v_o$  the voltage of interest. Identify

1. the system boundary,
2. the input vector,
3. the output vector,
4. a state vector,
5. an elemental equation,
6. and which equations might be continuity or compatibility equations.



re: a  
state-  
determined  
system



**Figure sdet.1:** illustration of a system and its environment.

