01.2 intro.sdet State-determined systems

1 A **system** is defined to be a collection of objects and their relations contained within a **boundary**. The collection of those objects that are external to the system and yet interact with it is called the **environment**. **System variables** are variables that represent the behavior of the system, both those that are internal to the system and those that are external—that is, with the system's environment.

2 There are three important classes of system variable, all typically expressed as vector-valued functions of time t, conventionally all expressed as column-vectors (and called "vectors" even though they're vector-valued functions …because nothing makes sense and we're all going to die). Consider Figure sdet.1 for the following definitions. **Input variables** are system variables that do not depend on the internal dynamics of the system; for a system with r input variables, the "**input vector**" is a vector-valued function $\mathbf{u} : \mathbb{R} \to \mathbb{R}^r$. The environment prescribes inputs, making them *independent variables*. Conversely, **output variables** are system variables of interest to the designer; for a system with m output variables, the "**output vector**" is a vector-valued function $\mathbf{y} : \mathbb{R} \to \mathbb{R}^m$. Outputs may or may not directly interact with the environment. Finally, a minimal set of variables that define the internal state of the system are defined as the **state variables**; for a system with n state variables, the "**state vector**" is a vector-valued function $\mathbf{x} : \mathbb{R} \to \mathbb{R}^n$.

3 We consider a very common class of system: those that are **state-determined**, which are those for which (Rowell **and** Wormley, 1997)

1. a mathematical description,
2. the state at time t_0, called the **initial condition** $\mathbf{x}(t)|_{t=t_0}$, and
3. the input $\mathbf{u}$ for all time $t \geqslant t_0$

are necessary and sufficient conditions to determine $\mathbf{x}(t)$ (and therefore $\mathbf{y}(t)$) for all $t \geqslant t_0$.

4 The "mathematical description" of the system requires a set of primitive elements be assigned to represent its internal and external interactions. The equations derive from two key types of mathematical relationships:

1. the input-output behavior of each primitive element and
2. the topology of interconnections among elements.

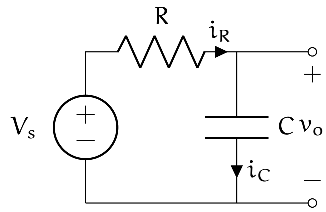
The former generate **elemental equations** and the latter, **continuity** or **compatibility equations**.

Example 01.2 intro.sdet-1

re: a state-determined system

In the RC circuit shown, let V_s be a source and v_o the voltage of interest. Identify

1. the system boundary,
2. the input vector,
3. the output vector,
4. a state vector,
5. an elemental equation,
6. and which equations might be continuity or compatibility equations.



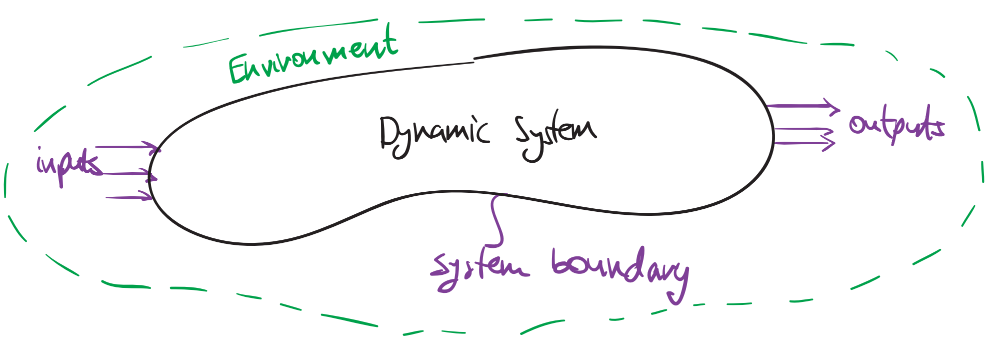


Figure sdet.1: illustration of a system and its environment.