01.4 intro.mecht Mechanical translational elements

1 We now introduce a few lumped-parameter elements for mechanical systems in translational (i.e. straight-line) motion. Newton's laws of motion can be applied. Let a **force** f and **velocity** v be input to a port in a mechanical translational element. Since, for mechanical systems, the power into the element is

$$\mathcal{P}(t) = f(t)v(t) \tag{1}$$

we call f and v the **power-flow variables** for mechanical translational systems. Some mechanical translational elements have two distinct locations between which its velocity is defined (e.g. the velocity across a spring's two ends) and other elements have just one (e.g. a point-mass), the velocity of which must have an inertial frame of reference. This is analogous to how a point in a circuit can be said to have a voltage—by which we mean "relative to ground." In fact, we call this mechanical translational inertial reference **ground**.

2 **Work** done on the system over the time interval $[0, T]$ is defined as

$$W \equiv \int_0^T \mathcal{P}(\tau)d\tau. \tag{2}$$

Therefore, the work done on a mechanical system is

$$W = \int_0^T f(\tau)v(\tau)d\tau. \tag{3}$$

3 The **linear displacement** x is

$$x(t) = \int_0^t v(\tau)d\tau + x(0). \tag{4}$$

Similarly, the **linear momentum** is

$$p(t) = \int_0^t f(\tau)d\tau + p(0). \tag{5}$$

4 We now consider two elements that can store energy, called **energy storage elements**; an element that can dissipate energy to a system's environment, called an **energy dissipative element**; and two elements that can supply power from outside a system, called **source elements**.

Translational springs

5 A **translational spring** is defined as an element for which the displacement x across it is a monotonic function of the force f through it. A **linear translational spring** is a spring for which Hooke's law applies; that is, for which

$$f(t) = kx(t), \tag{6}$$

where f is the force through the spring and x is the displacement across the spring, minus its unstretched length, and k is called the **spring constant** and is typically a function of the material properties and geometry of the element. This is called the element's **constitutive equation** because it constitutes what it means to be a spring.

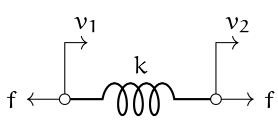

Figure mecht.1: schematic symbol for a spring with spring constant k and velocity drop $v = v_1 - v_2$.

6 Although there are many examples of nonlinear springs, we can often use a linear model for analysis in some operating regime. The **elemental equation** for a linear spring can be found by time-differentiating Equation 6 to obtain

We call this the elemental equation because it relates the element's power-flow variables f and v.

7 A spring stores energy as elastic potential energy, making it an *energy storage element*. The amount of energy it stores depends on the force it applies. For a linear spring,

$$\mathcal{E}(t) = \frac{1}{2k} f(t)^2. \tag{7}$$

Point-masses

8 A non-relativistic translational point-mass element with mass m, velocity v (relative to an inertial reference frame), and momentum p has the constitutive equation

$$p = mv. \tag{8}$$

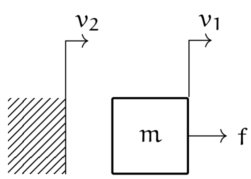

Figure mecht.2: schematic symbol for a point-mass with mass m and velocity drop $v = v_1 - v_2$, where v_2 is the constant reference velocity.

Once again, time-differentiating the constitutive equation gives us the elemental equation:

which is just Newton's second law.

9 Point-masses can store energy (making them *energy storage elements*) in gravitational potential energy or, as will be much more useful in our analyses, in kinetic energy

$$\mathcal{E}(t) = \frac{1}{2} m v^2. \tag{9}$$

Dampers

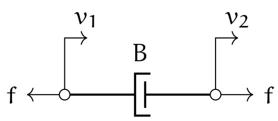

Figure mecht.3: schematic symbol for a damper with damping coefficient B and velocity drop $v = v_1 - v_2$.

10 **Dampers** are defined as elements for which the force f through the element is a monotonic function of the velocity v across it. **Linear dampers** have constitutive equation (and, it turns out, elemental equation)

$$f = Bv \tag{10}$$

where B is called the **damping coefficient**. Linear damping is often called **viscous damping** because systems that push viscous fluid through small orifices or those that have lubricated sliding are well-approximated by this equation. For this reason, a damper is typically schematically depicted as a **dashpot**.

11 Linear damping is a reasonable approximation of lubricated sliding, but it is rather poor for **dry friction** or **Coulomb friction**, forces for which are not very velocity-dependent. Aerodynamic **drag** is quite velocity-dependent, but is rather nonlinear, often represented as

where c is called the drag constant.

12 Dampers dissipate energy from the system (typically to heat), making them *energy dissipative elements*.

Force and velocity sources

13 An **ideal force source** is an element that provides arbitrary energy to a system via an independent (of the system) force. The corresponding velocity across the element depends on the system.

14 An **ideal velocity source** is an element that provides arbitrary energy to a system via an independent (of the system) velocity. The corresponding force through the element depends on the system.