02.3 graphs.connect Element interconnection laws

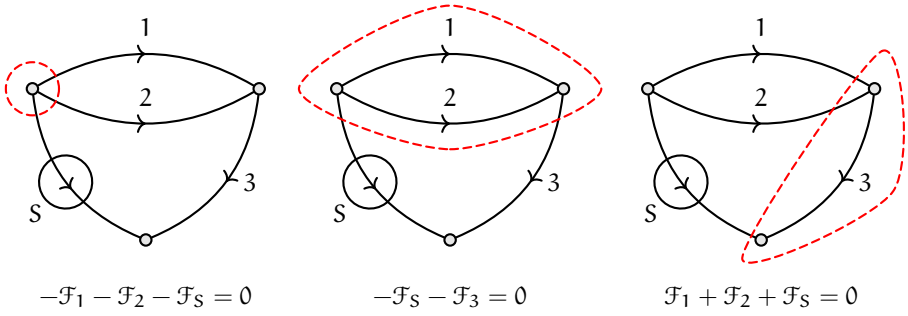
1 The interconnections among elements constrain across- and through-variable relationships. The first element interconnection law requires the concept of a **contour** "⬭": a closed path that does not self-intersect superimposed over the linear graph. The first interconnection law is called the **continuity law**.

Definition 02 graphs.2: continuity law

The sum of the through-variables that flow on *into* a contour on a linear graph is zero, or, in terms of generalized through-variables $\mathcal{F}_i$ for N elements with through variables defined as positive into the contour,

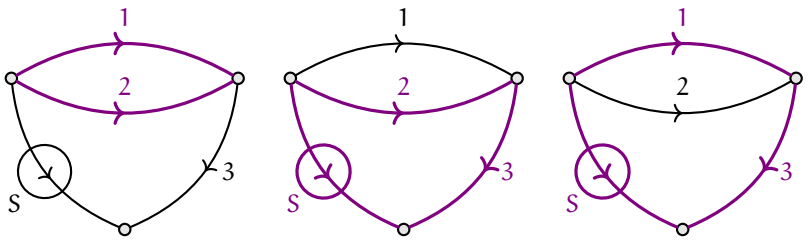
$$\sum_{i=1}^{N} \mathcal{F}_i = 0. \tag{1}$$

2 Contours can enclose any number of nodes and edges, as illustrated in Figure connect.1. **Kirchhoff's current law** (KCL) is the special case of the continuity law for electronic systems.



$-\mathcal{F}_1 - \mathcal{F}_2 - \mathcal{F}_S = 0$ $\quad -\mathcal{F}_S - \mathcal{F}_3 = 0$ $\quad \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_S = 0$

Figure connect.1: illustration of different contours, denoted with red dashed lines "⬭," contours for which the continuity law applies, as shown below each graph.



$v_1 - v_2 = 0$ $v_2 + v_3 - v_S = 0$ $v_1 + v_3 - v_S = 0$

Figure connect.2: illustration of different loops, denoted with violet edges "⌒," loops for which the compatibility law applies.

3 The second interconnection law we consider requires the concept of a **loop** "⌓": a continuous series of edges that begin and end at the same node, not reusing any edges.[2] The second interconnection law is called the **compatibility law**.

Definition 02 graphs.3: compatibility law

The sum of the across-variable drops on edges around any closed loop on a linear graph is zero, or, in terms of generalized across variables v_i for N elements in a loop,

$$\sum_{i=1}^{N} v_i = 0. \tag{2}$$

A loop can be "inner" or "outer," as shown in Figure connect.2. **Kirchhoff's voltage law** (KVL) is the special case of the compatibility law for electronic systems.

Example 02.3 graphs.connect-1

re: element interconnection laws

For the system shown, (a) write three unique continuity and three unique compatibility equations. Moreover, (b) write a continuity equation solved for f_4 in terms of f_S and f_1. Finally, (c) write a compatibility equation

[2] Technically, we need not restrict the definition to series that do not reuse edges for purposes of the compatibility law, but these loops are superfluous and we exclude them here.

solved for v_5 in terms of v_S, v_3, and v_4.